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Calculating temperature dependence over long time periods: a comparison and study of methods

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Abstract

Nonlinear temperature dependence plays a major role in a large variety of ecological models. For the sake of simplicity and efficiency, the temperature dependence functions in many models are calculated for monthly or yearly time intervals, using temperature means or interpolations between means as input. As a consequence, information about the variability of the temperature input data is lost, which leads to a bias in the temperature dependence function and to errors in the model results. We tested the performance of a range of methods against this common approach for calculating temperature dependence on a larger time scale, i.e. for a temporal aggregation. The methods estimate the expected value of the dependence function in different ways, using the mean or standard deviation of temperature variables in different temporal resolutions as input. In our tests we used temperature dependence functions from four different ecological fields; hourly temperature data sets from various climatically differing sites were used as input. The precision of the tested methods increased with the resolution of the input data, although computing time increased. The mean errors ranged from less than 1% to about 8% for the aggregation to 1 month and from about 1% to over 30% for the aggregation to 10 years. The most precise and efficient method is the explicit calculation of the expected value for the dependence function, which is based on the mean and standard deviation of hourly temperatures. The least precise but most efficient method is the common application of the dependence function to mean values. The quality of these methods is mainly determined by the quality of the approximation of the temperature variability. Condensing highly resolved input data into means is only appropriate if either the dependence functions are linear in the observed temperature range, or low precision but very high computing efficiency is required. Given a certain requirement on precision or computing efficiency, we are now able to indicate for a number of input data resolutions the appropriate method to calculate temperature dependence over long time periods. © 1997 Elsevier Science B.V.

Keywords: Temperature dependence; Biological processes; Modelling; Temporal aggregation

1. Introduction

Many biologically, ecologically, and agriculturally relevant processes are controlled by temperature dependent rates. The functions with which these

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rates depend on temperature T, can have various, usually nonlinear shapes, e.g. Gaussian or exponential. In the following such temperature dependence functions and their approximations are denominated by $\varphi(T)$. The accumulated effect of temperature, e.g. on insect maturity or plant phenology, is normally measured by means of the integral

$$M(t,t_0) := \int_{t_0}^{t} \varphi(T(\tau))\mathrm{d}\tau \qquad (1)$$

over a time interval (t_0,t). This integral is often referred to as 'physiological time', 'day-degree-sum', or 'heat-unit-sum'.

Temperature dependence also plays a major role in many ecological simulation models, ranging from pest prognosis models, such as BUGOFF2 (Blago and Dickler, 1990) and APFWICK (Lischke and Blago, 1990; Lischke, 1992), and crop phenology models, such as BIOTIME (Kirsta and Tarabrin, 1994), to models examining the sensitivity of ecosystems to a potential climatic change, such as the forest patch models FORSKA (Prentice et al., 1993), FORCLIM (Fischlin et al., 1995; Bugmann, 1996) and DisCForM (Lischke et al., 1997a), where physiological time determines the growth and thus competition and succession of trees.

The precision of the temperature dependence function can have a crucial influence on the outcome of such models, depending on the model sensitivity to the temperature dependence function. The most exact approach is to calculate $M(t,t_0)$ (Eq. (1)) by summing the actual values of the dependence function using temperature data at high temporal resolution, which reflect the diurnal and even more frequent temperature fluctuations.

However, due to practical constraints such as the lack of appropriate input data or long computation times, in many models a larger time step is chosen, and temperature dependence is calculated by applying the temperature dependence function either to mean temperatures (e.g. monthly temperature means as in FORCLIM), or to an interpolated temperature course (e.g. in FORSKA or in the BIOTIME-model). Yet, monthly or yearly temperature means or interpolations between means do not contain complete information about temperature variability in the considered period, particularly not about the intra-daily variability. If the dependence function is nonlinear, which is realistic for many cases, such a simple approach can lead to a loss of precision in the model outcome.

One commonly used approach for calculating temperature dependence over long intervals, i.e. aggregating it, is to replace the nonlinear dependence function by a linear one. But this approximation again can lead to a considerable loss of precision (cf. Blago and Dickler, 1990). On the other hand, empirical correction functions of the temperature dependence as used by Bugmann (1994) confine the model application to the regions where those functions have been estimated.

Thus, there is a need for methods which aggregate temperature dependence functions over long periods in a precise and efficient way. A range of methods for such a temperature dependence aggregation has been compiled and developed (cf. Lischke et al., 1997b). The common principle of the methods is that they estimate the expected value of the temperature dependence function over the aggregation period. They take into account the information about temperature variability contained in the available input data, are applicable for general, i.e. nonlinear temperature dependence functions, and suitable for input data of different resolution.

In the present paper, we tested the precision and computing efficiency of these aggregation methods in four case studies against the commonly used application of the dependence function to temperature means (method DA) and to temperatures approximated by the sine-method of Allen (1976) (method ALLEN), or to temperatures produced by a random weather generator (method STOCH). We assessed in particular, how much the assumptions and approximations underlying the methods affect their precision.

The temperature dependence functions used in the case studies cover different ecological fields and hierarchical levels. For example, for temperature driven development, such as insect maturing or plant growth, we examine the development of the codling moth (*Cydia pomonella*). Temperature dependent timing processes, such as insect diapause (e.g. overwintering) or seed vernalisation, are represented by the chilling requirement of the apple tree bud rest break. Net photosynthesis of trees is an example of an aggregated physiological process; net photosynthesis rates of several tree species are considered separately to assess the influence of the temperature dependence aggregation on interspecific competition. With soil respiration an ecosystem process is consid-

Table 1
Overview of the temperature dependence aggregation methods. They are divided according to the type of method (Explicit expectation value calculation; Stochastic expectation value calculation or or A, dependence function of average input), the resolution and kind of input data needed (T, hourly temperature; T_{min},T_{max}, daily temperature extrema; Δ, daily temperature amplitude; $\bar{T}$, daily temperature mean; $\bar{T}_m$, approximated daily temperature mean; $\mu_{\bar{T}}$, monthly mean temperature; μ_Δ, monthly mean amplitude), the statistical parameters estimated from these data (μ, mean; σ, standard deviation) and the approximations used (φ, dependence function; TC, daily temperature course; ND, normal distribution)

Method	Abbrev.	Type	Temperature input data			Approx.
			Temporal resolution	Variables	Stat. param.	
Stochastic generation of average dependence function	STOCH	S	hours	T	μ_T,σ_T	φ
Expectation value of dependence function of hourly temperatures	EDH	E	hours	T	μ_T,σ_T	φ TC
Expectation value of dependence function of hourly temperatures approximated by triangle of mean and amplitude	EDHT1	E	days	$\bar{T}$ $\Delta = T_{max} - T_{min}$	$\mu_{\bar{T}},\sigma_{\bar{T}}$ μ_Δ,σ_Δ	φ TC
Expectation value of dependence function of hourly temperatures approximated by triangle based on extrema	EDHT2	E	days	$\bar{T}_m = \frac{T_{min}+T_{max}}{2}$ $\Delta = T_{max} - T_{min}$	$\mu_{\bar{T}_m}, \sigma_{\bar{T}_m}$ μ_Δ,σ_Δ	φ TC
Sine-sine method of Allen	Allen	–	days	$\bar{T}_m = \frac{T_{min}+T_{max}}{2}$ $\Delta = T_{max} - T_{min}$	–	–
Expectation value of dependence function of daily temperature triangle based on mean and amplitude	EDDT1	E	days	$\bar{T}$ $\Delta = T_{max} - T_{min}$	$\mu_{\bar{T}},\sigma_{\bar{T}}$ $\bar{\sigma}_T$ μ_Δ,σ_Δ	φ TC ND
Expectation value of dependence function of daily temperature triangle of extrema	EDDT2	E	days	$\bar{T}_m = \frac{T_{min}+T_{max}}{2}$ $\Delta = T_{max} - T_{min}$	$\mu_{\bar{T}_m},\sigma_{\bar{T}_m}$ μ_Δ,σ_Δ	φ TC ND
Expectation value of dependence function of daily temperature mean	EDM	E	days	$\bar{T}$	$\mu_{\bar{T}},\sigma_{\bar{T}}$	φ
Dependence function of average daily temperature triangle	DAT	A	months	$\mu_{\bar{T}},\mu_\Delta,$	$\mu_{\bar{T}},\mu_\Delta$	φ TC
Dependence function of average temperature	DA	A	months	$\mu_{\bar{T}} = \mu_T$	$\mu_{\bar{T}}$	φ

ered, which integrates over space and many different organisms.

2. Tested methods

The methods tested in this study calculate the integral of Eq. (1) by the expected value $E[\varphi(T)]$ of the dependence function $\varphi(T)$ of the hourly temperatures in the aggregation interval (t,t_0), multiplied by its length $t-t_0$ by

$$M(t,t_0):=(t-t_0)\cdot E[\varphi(T)]$$

The methods differ in how the expected value $E[\varphi(T)]$ is calculated or approximated, particularly in how the variability of the temperature data is estimated. Table 1 gives an overview of the different methods, which are divided using the following criteria.

Three different types of approaches are used to determine the expected value $E[\varphi(T)]$:

1. The expected value is determined stochastically (type S) by sampling 1000 temperature realisations from a normal distribution (Monte Carlo simulation), based on mean and standard deviation of the hourly temperature μ_T and σ_T, calculating the temperature dependence of each and averaging it.
2. The dependence function is applied to the mean of the regarded input variable(s) (e.g. mean hourly temperature in method DA or mean daily temperature triangle in method DAT) in the aggregation period (type A).
3. The expected value of the dependence function φ is calculated explicitly (type E), assuming the temperature variable X to be normally distributed with the density function p_x. For example, in method EDH the expected value is given by

$$E[\varphi(T)]=\int_{-\infty}^{\infty}\varphi(x)\cdot p_T(x)\mathrm{d}x$$

The input data required by the methods are the statistical parameters mean μ, or mean and standard deviation σ of temperature variables, which are given at hourly, daily, or monthly resolution. As variables the hourly temperature T, the daily temperature extremes T_{max} and T_{min}, the daily temperature amplitude Δ, the daily temperature mean $\bar{T}$ (either measured such as in EDHT1, EDDT1, and EDM or approximated by $\bar{T}_m=(T_{min}+T_{max})/2$ such as in the Allen method, EDHT2, and EDDT2), the monthly mean temperature $\mu_{\bar{T}}$, or the monthly mean of the daily temperature amplitude μ_Δ are used.

1. Methods EDH, EDHT1, and EDHT2 determine the expected value of the hourly dependence function based on the mean and standard deviation of the hourly data. These statistical parameters are either estimated from the input data in method EDH, or derived from the means and standard deviations of the daily temperature mean $\bar{T}$ or $\bar{T}_m$, and daily temperature amplitude Δ (based on the assumption of a triangle shaped daily temperature

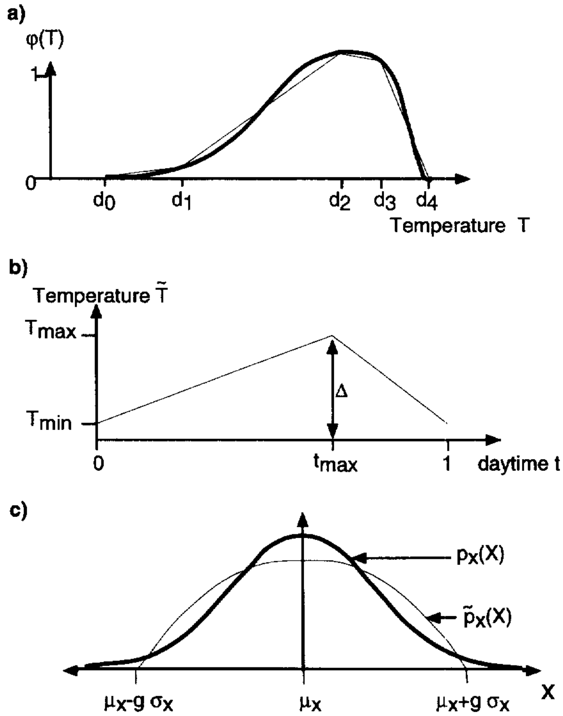

Fig. 1. Approximations (hairlines) used in aggregation methods. (a) Approximation of temperature dependence function $\varphi(T)$. Thick line, exact dependence function; hairline, approximation. (b) Approximation of daily temperature course $\tilde{T}(t)$. T_{min}, T_{max}, temperature extremes; t_{max}, time when temperature maximum is reached; Δ, temperature amplitude. (c) Approximation of density function of normal distribution ($p_x(x)$, thick line) by a parabola ($\tilde{p}_x(x)$, hairline). μ_X, mean; σ_X, standard deviation of temperature; g, slope parameter of parabola ($g=2.3$ in simulations).

course (cf. Fig. 1(b)) in methods EDHT1 and EDHT2).

2. Methods EDDT1, EDDT2, and EDM determine the expected value of the daily dependence function $\nu(\bar{T},\Delta)$. The expected value, e.g. of method EDDT1, is

$$E\left[\nu(\bar{T},\Delta)\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\nu(y,z)\cdot p_{\bar{T}}(y)\,\mathrm{d}y\,p_{\Delta}(z)\,\mathrm{d}z \qquad (2)$$

In method EDM, $\nu(\bar{T},\Delta)$ is obtained by applying the dependence function to the daily mean temperature $\bar{T}$. In methods EDDT1 and EDDT2, $\nu(\bar{T},\Delta)$ is given by the integral over the daily course of φ, which is obtained by a triangle-shaped approximation of the daily temperature course, determined by the two variables temperature mean $\bar{T}$ or $\bar{T}_m$, and amplitude Δ.

Some methods rely on assumptions and approximations, e.g. to calculate the expected values $E[\varphi(T)]$ and $E[\nu(\bar{T},\Delta)]$, which is not explicitly possible for each dependence function $\varphi(T)$ or density function p_x. Also, input parameters are approximated, e.g. temperature mean in EDDT2 and EDHT2, to handle input data of insufficient resolution.

The random variables daily temperature mean $\bar{T}$ and $\bar{T}_m$, and the amplitude Δ are assumed to be independent. As approximations we use a piece-wise linear function for the nonlinear temperature dependence function $\varphi(T)$ (cf. Fig. 1(a)), an asymmetric triangle $\tilde{T}(t)$ with the same minimum temperature at the beginning and end of the day and a variable time point t_{max} of the maximum temperature for the daily temperature course (TC) (cf. Fig. 1(b)), and a parabola $\tilde{p}$ (cf. Fig. 1(c)) for the density functions $p_{\bar{T}}(y)$ and $p_{\Delta}(z)$ of the normal distribution in Eq. (2).

We used the approximated dependence function applied to the hourly temperature values and averaged over each test period as a reference (exact value) in the case study test. For comparison we utilised the commonly used methods of applying the dependence function either to the mean temperature, i.e. method DA, or to hourly temperatures obtained by a sine-wave approximation of the daily temperature course (Allen, 1976) (without empirical correction) as well as the stochastic temperature generator STOCH.

3. Case study tests

To test the precision and computing efficiency of the seven new aggregation methods and three comparison methods DA, STOCH, and Allen, four case study tests with the following temperature dependent processes from different ecological fields were carried out:

1. soil respiration (cf. Fig. 2(a)), modelled with the function used in Parton et al. (1993);
2. bud rest break (cf. Fig. 2(b)) of apple trees, as modelled by Del Real-Laborde et al. (1990);
3. tree net photosynthesis (cf. Fig. 2(c)) of the tree species *Pinus cembra*, *Picea abies*, *Abies alba*, *Larix decidua*, *Betula pendula*, and *Fagus silvatica*, parameterised with the cardinal temperature values documented in Pisek et al. (1969) (cf. Table 2);
4. development of the larval and pupal stages of the codling moth (*Cydia pomonella* L. (Lepidoptera, Tortricidae)) with the temperature dependence function from Lischke and Blago (1990) (cf. Fig. 2(d)).

For the soil respiration case study an 8-month time series (data provided by Richner, 1994) of hourly soil temperatures at 10 cm depth under grass was used as input. For case studies 2, 3 and 4, 11 years of hourly air temperature from six climatically differing sites (ranging from dry temperate to boreal climate) in the European Alps served as input data. For the bud rest break study, only temperature data of the winter months (October–April) were included in the analysis.

We calculated the monthly expected values of the dependence functions, i.e. aggregated them from an hourly to a monthly time step in case studies 1, 2 and 3. We also determined the mean values of the different estimators for temperature mean and variability used in the examined methods.

To explore the influence of the aggregation period length, we used the codling moth development as example and determined the 2-, 3- and 6-monthly, yearly, and 10-yearly expected values. Considering codling moth development over such long time periods can give information about the potential number of codling moth generations per year. We also determined the mean values of the different estimators for temperature mean and variability used in the examined methods.

Precision and computing efficiency of all methods were determined for each aggregation period P_a at each test site. The precision, i.e. the error due to the aggregation, was measured as difference between exact expected value of temperature dependence and the value obtained by the aggregation methods, in percent of the average exact value. The computing efficiency ξ of the methods is given in percent computing time Δt_{method} of the run-time needed to calculate the exact value, which is given by the P_a fold of the time needed for 1 day

$$\xi = \frac{100 \cdot \Delta t_{method}}{\Delta t_{exact}} = \frac{100 \cdot \Delta t_{method}}{P_a \cdot \Delta t_{exact,day}}$$

Table 2
Parameter values (°C) of the temperature dependence function for tree photosynthesis (Pisek et al., 1969)

Species	T_{min}	$T90_{min}$	T_{opt}	$T90_{max}$	T_{max}
Abies alba	−3.5	8	15	22	38
Picea abies	−4	11	14	19	37
Pinus cembra	−5.5	9	12.5	18.5	36
Larix decidua	−3	11	16.5	21	38
Betula pendula	−3	12	17	21	42
Fagus silvatica	−6.5	13	18	23	43

The time for calculation of the statistical parameters of the data was not included in this measurement.

All methods except the Allen method need only one evaluation regardless of the aggregation period length. The ratio $\Delta t_{method}/\Delta t_{exact,day}$ of the time required for this single evaluation to the time re-

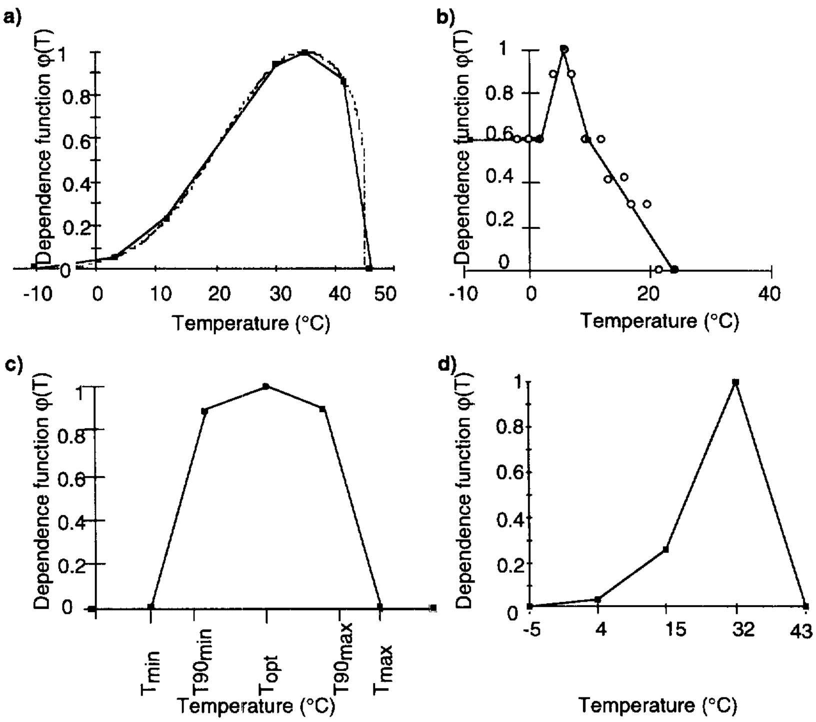

Fig. 2. Temperature dependence functions of the processes tested in the case studies. Dotted line or separate points, dependence function; solid line, approximation; black squares, discretisation points of approximation. (a) Soil respiration. (b) Apple tree bud rest break. (c) Tree net photosynthesis (for species specific parameter values see Table 2). (d) Codling moth larval and pupal development.

quired for the exact evaluation for 1 day can then be determined by

$$\frac{\Delta t_{\text{method}}}{\Delta t_{\text{exact,day}}} = \frac{\xi \cdot P_{\text{a}}}{100}$$

This ratio also yields the critical aggregation period length $P_{\text{a,crit}}$, above which the tested methods starts to be faster than the exact evaluation.

The performance tests were implemented in the programming language Modula-2 using the program library of the Dialog-Machine V2.2 (Fischlin, 1991) and run on a SUN SPARCserver10 with the batch environment RASS V1.1 (Thöny et al., 1994). Analytical examinations of the functions were performed with the help of the symbolic calculation software MATHEMATICA.

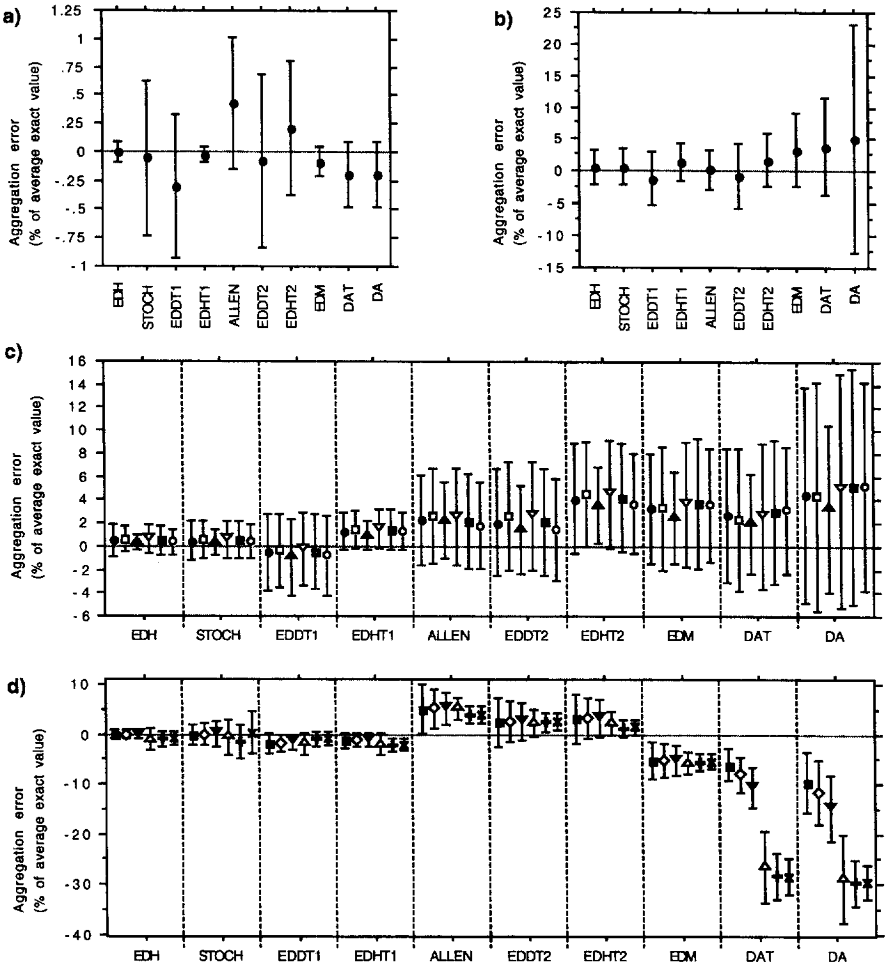

Fig. 3. Aggregation errors: differences between temperature dependence mean calculated with measured temperatures (exact) and expected values calculated with studied methods. The differences are expressed as a percentage of the exact temperature dependence averaged over all years and sites. Means and standard deviations of the difference distributions are given as point symbols and bars. Dependence functions of: (a) soil respiration; (b) apple bud rest break; (c) tree net photosynthesis (tree species: •, *Abies alba*; □, *Betula pendula*; ▲, *Fagus silvatica*; ▽, *Larix decidua*; ■, *Picea abies*; ○, *Pinus cembra*); (d) codling moth development. Aggregation periods were 1 month in (a), (b) and (c); in (d) ■, 1 month; ◇, 2 months; ▼, 3 months; △, 6 months, +, 1 year; ×, 10 years.

4. Results

Fig. 3(a)–(c) show the errors of the different approaches for the aggregation to 1 month in three case studies. For the soil respiration case study the errors are very small, below 0.5%. In contrast to this, the aggregation errors in the case studies for apple tree bud rest break and tree net photosynthesis reach up to 30% for method DA, with the mean values ranging from below 1% for method EDH to 12% for method DA. The absolute mean errors and the error variability increase from EDH to DA, as do the differences of the errors between the tree species in the tree net photosynthesis study. Some methods calculate the expected value by including information about the daily temperature variability, by using either the temperature variance as EDH and STOCH, or the temperature amplitude as EDHT1, EDHT2, EDDT1, and EDDT2. These methods are of a similar or higher precision as the sine-wave-approximation of Allen (1976), which also uses the temperature amplitude as input.

In Fig. 3(d) (codling moth case study) the errors for the aggregation to various periods are compared. For the 1 month aggregation the results are similar to those of case studies 2 and 3. As the period length increases, in methods DA and DAT the absolute mean errors increase up to 30%, whereas in the other methods it remains constant. Fig. 4 shows for the same case study the computation time of the different methods measured in percent of the time needed for the exact calculation. The computation time was plotted against the mean percentage aggregation error.

Fig. 5 qualitatively summarises the relationship between precision, computing efficiency, aggregation period length and input data resolution. The general trend is that precision increases with the

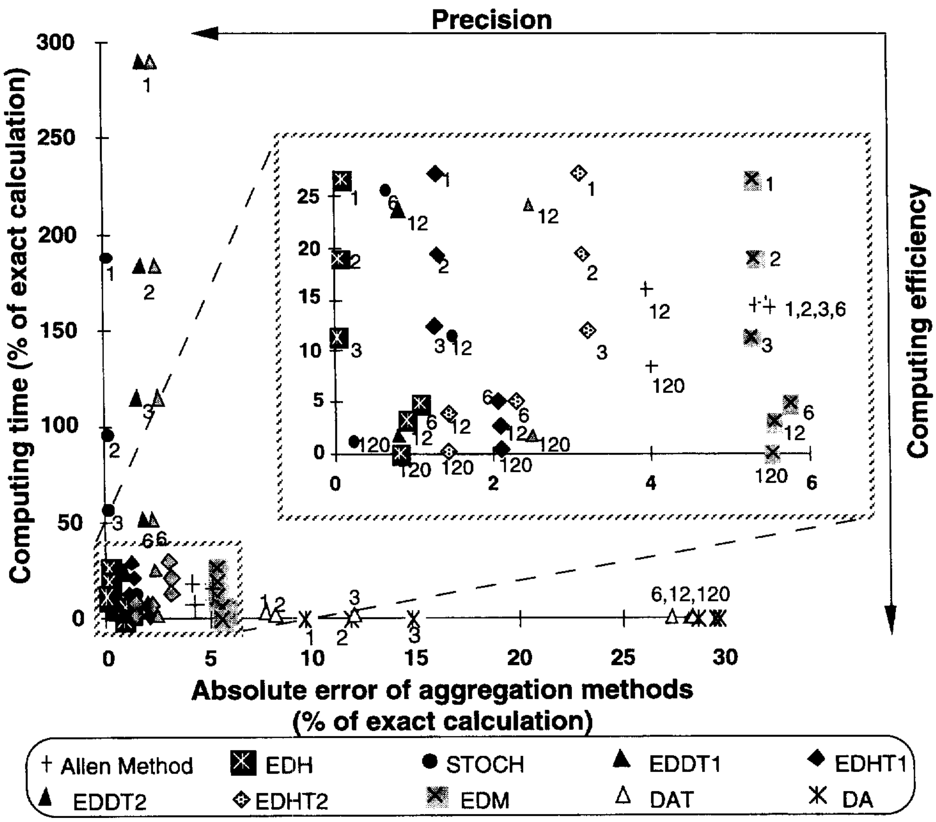

Fig. 4. Efficiency measured as computing time vs. precision measured as mean aggregation error (see also legend to Fig. 3). The small numbers near the symbols refer to the length of the aggregation period. The dotted rectangle zooms into the region near (0,0).

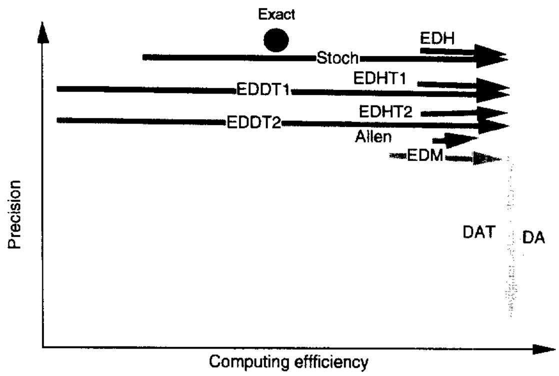

Fig. 5. Qualitative scheme of the precision of the aggregation methods depending on input data resolution, computing efficiency and aggregation interval. The arrows symbolise the aggregation interval lengths, with 1 month at the beginning and 10 years at the end of the arrow. The input data resolution is represented by the grey scale of the arrows, which ranges from black for the hourly input data to very light grey for the monthly mean input.

information contained in the input data. At the same time computing efficiency decreases. In Figs. 4 and 5 we can distinguish between three groups of methods.

1. Methods EDH, EDHT1, EDHT2, Allen, and EDM combine high precision (error less than 5.5%) with high computing efficiency (from below 1% to 30% of the exact computation) for all tested aggregation periods. The use of the Allen method pays off for all aggregation periods longer than 1 day, the use of the other four methods for aggregation periods longer than 10 days.
2. Methods EDDT1, EDDT2, and STOCH have mainly the same precision as group (1), but 9, 9, and 5 times longer computing times, respectively. This means that these methods start to pay off at aggregation periods of 90, 90, and 50 days, respectively.
3. Methods DA and DAT are the most efficient (less than 1% for all aggregation periods, always faster than exact calculation), but least precise methods. Their precision decreases with the aggregation error increasing from over 8% to 30% with the aggregation period length.

To examine the differences in the outcome of the various methods we studied how the results depend on the mean monthly temperatures. Fig. 6 shows the deviations of method DA in the codling moth and photosynthesis study (*Abies alba*, aggregation period 1 month) vs. temperature means. The highest absolute errors are found close to the grid points of the approximated dependence functions, e.g. at 4°C and 15°C for the codling moth study and at −3.5°C, 8°C and 15°C for the photosynthesis study. This pattern was also found for the other methods except for ALLEN and STOCH, though not so clearly. All methods underestimate the true expected value of the dependence function when the dependence function is concave, and overestimate it when the function is convex. The error increases with the angle between the two linear parts of the approximated dependence function. The absolute errors increase also with increasing standard deviation for the same temperature mean.

The tested methods include various assumptions about the distribution of the temperature data (Fig. 7) and estimate the parameters of these distributions in different ways.

The temperature data could not be considered to be strictly normally distributed in more than 12% of all tested months (chi-square test (Sachs, 1984), $\alpha = 0.05$).

We determined the mean differences of the estimators of temperature mean and standard deviation to the measured temperature means and standard deviations (Table 3). The temperature distributions, which are defined by these variability and mean estimators, are shown in Fig. 7. Most methods deviate only slightly from the normal distribution with measured mean and standard deviation (bold line). In methods EDDT2 and EDDH2 the mean temperatures are slightly overestimated (Table 3, Fig. 7), because

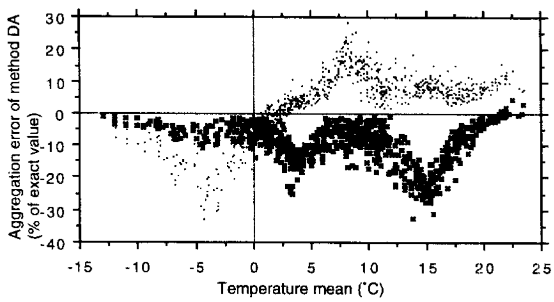

Fig. 6. Aggregation error (see also legend to Fig. 3) vs. mean temperature. Large dots, codling moth development; small dots, net photosynthesis of *Abies alba*.

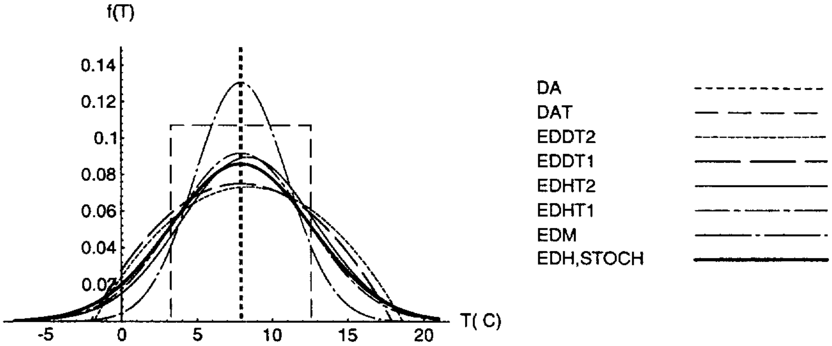

Fig. 7. Temperature distributions assumed in the tested methods, based on approximations of distribution type and on mean values of temperature mean and variability estimates (Table 3).

they are based on a daily mean estimated from the daily extremes. Methods DA, DAT, and EDM show larger deviations. For DAT this is partly due to the uniform distribution of temperature which is implicitly assumed by the average daily temperature triangle used in this method. Furthermore, DA does not take into account variability at all and methods DAT and EDM use either only intra-daily or inter-daily variability. This leads to an underestimation of the temperature variability. Inter-daily temperature variability as measured by the standard deviation $\sigma_{\bar{T}}$ of the daily mean $\bar{T}$ increases from 1 to 6 months aggregation period length. Intra-daily variability as measured by the average daily standard deviation $\bar{\sigma}_T$ remains constant (Table 4). This is reflected by the mean differences between the two variabilities and also by the differences in the outcome of methods EDM and DAT (Fig. 3(d)).

The relations between precision of the methods and variability estimation can be explained by considering the expected value of a dependence function $\varphi(T)$ which consists of two linear parts (e.g. the second and third part in Fig. 2(d), with $T_i = 15$ and $\varphi_i = 0.25$). These linear parts are separated by the grid point (T_i, φ_i). The dependence function can be expressed by

$$\varphi(T) = \begin{cases} \varphi + a \cdot (T - T_i) & \text{for } T < T_i \\ \varphi + (a + \partial) \cdot (T - T_i) & \text{for } T > T_i \end{cases}$$

where a is the slope of the first, and $(a + \partial)$ the slope of the second linear part; the difference ∂ of

Table 3
Average deviation of estimators of temperature variability from measured temperature standard deviation (estimated − exact)

Method	Assumed distribution	Deviations of estimators (°C)	
		μ_T	σ_T
DA	None	0	No variability considered
DAT	Rectangle distribution between T_{min} and T_{max}	0	$\bar{\Delta}/2 = 0.02$
EDM	Normal distribution	0	−1.59
EDHT1	Normal distribution	0	−0.30
EDHT2	Normal distribution	0.46	−0.19
EDDT1	Normal distribution approximated by parabola	0	−0.30
EDDT2	Normal distribution approximated by parabola	0.46	−0.19
EDH	Normal distribution	0	0
STOCH	Normal distribution	0	0

Table 4
Differences between inter-daily variability ($\sigma_{\bar{T}}$) and intra-daily variability over all sites, years and aggregation periods 1–6 months

Aggr. period	$\bar{\sigma}_T$		$\sigma_{\bar{T}}$		$\bar{\sigma}_T - \sigma_{\bar{T}}$
	Mean (°C)	SD (°C)	Mean (°C)	SD (°C)	Mean (°C)
1	3.36	0.99	3.05	0.99	0.31
2	3.40	0.90	3.60	0.96	−0.20
3	3.41	0.87	4.31	1.05	−0.90
6	3.44	0.71	7.21	0.91	−3.77

the two slopes is a measure for the angle between the linear parts, i.e. for the curvature. If it is positive, the dependence function is concave, if it is negative the function is convex. The expected value of $\varphi(T)$ is

$$\begin{aligned}
E[\varphi(T)] &= \int_{-\infty}^{\infty} \varphi(T)\cdot p_T(T)\mathrm{d}T \\
&= \varphi(T_i) + a\int_{-\infty}^{T_i}(T-T_i)\cdot p_T(T)\mathrm{d}T \\
&\quad + (a+\partial)\int_{T_i}^{\infty}(T-T_i)\cdot p_T(T)\mathrm{d}T \\
&= \varphi(T_i) + a\int_{-\infty}^{\infty}(T-T_i)\cdot p_T(T)\mathrm{d}T \\
&\quad + \partial\int_{T_i}^{\infty}(T-T_i)\cdot p_T(T)\mathrm{d}T \\
&= \varphi(T_i) + a\cdot(\mu_T - T_i) \\
&\quad + \underbrace{\partial\int_{T_i}^{\infty}(T-T_i)\cdot p_T(T)\mathrm{d}T}_{\phi(\sigma)}
\end{aligned} \tag{3}$$

with the density function $p_T(T)$ of the normal distribution. The empirical results of the case studies suggest that the calculated expected values decrease with decreasing variability. In Eq. (3), variability is contained only as standard deviation σ of the distribution density function $p_T(T)$ in the integral $\phi(\sigma)$. The solution of this integral is (μ: mean of the temperature distribution; erf: error function (Bronstein et al., 1995))

$$\begin{aligned}
\phi(\sigma) &= \partial\int_{T_i}^{\infty}(T-T_i)\cdot\frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{(T-\mu)^2}{2\sigma^2}}\mathrm{d}T \\
&= \frac{\partial}{\sqrt{2\pi}}\left(-\sigma\cdot\mathrm{e}^{-\frac{(T-\mu)^2}{2\sigma^2}}\right. \\
&\quad \left. + \sqrt{\frac{\pi}{2}}\cdot(\mu-T_i)\cdot\mathrm{erf}\left(\frac{(T-\mu)}{\sqrt{2}\sigma}\right)\right)\Bigg|_{T_i}^{\infty} \\
&= \frac{\partial}{\sqrt{2\pi}}\left(\sqrt{\frac{\pi}{2}}\cdot(\mu-T_i) + \sigma\cdot\mathrm{e}^{-\frac{(T_i-\mu)^2}{2\sigma^2}}\right. \\
&\quad \left. - \sqrt{\frac{\pi}{2}}\cdot(\mu-T_i)\cdot\mathrm{erf}\left(\frac{(T_i-\mu)}{\sqrt{2}\sigma}\right)\right) \\
&= \frac{\partial}{\sqrt{2\pi}}\left(\sqrt{\frac{\pi}{2}}\cdot(\mu-T_i)\cdot\left(1-\mathrm{erf}\left(\frac{(T_i-\mu)}{\sqrt{2}\sigma}\right)\right)\right. \\
&\quad \left. + \sigma\cdot\mathrm{e}^{-\frac{(T_i-\mu)^2}{2\sigma^2}}\right)
\end{aligned}$$

The derivative of $\phi(\sigma)$ with respect to σ is

$$\frac{\mathrm{d}\phi(\sigma)}{\mathrm{d}\sigma} = \frac{\partial}{\sqrt{2\pi}}\mathrm{e}^{-\frac{(T_i-\mu)^2}{2\sigma^2}}$$

This derivative, describing the change of $\phi(\sigma)$ and therefore of the expected value with respect to σ, depends on the sign and value of curvature parameter ∂, on the distance between temperature mean μ and grid point T_i, and on σ. If σ is estimated too small and the dependence function is concave, i.e. ∂ positive, the expected value is underestimated. This effect increases when mean μ approaches grid point T_i, such as also found in the empirical studies. Equivalently, the expected value is overestimated if the dependence function is convex, i.e. ∂ negative.

5. Discussion and conclusions

In this study a range of approaches for aggregating temperature dependence functions to longer time periods, i.e. to determine long term expected values of dependence functions, was tested. The aim was to understand the behaviour of the methods and to assess the suitability of the methods for a variety of input data resolutions, given precision and computing efficiency requirements.

The quality of the methods depends on (1) the estimation of the temperature distribution and (2) the approximation of the dependence function with piece-wise linear functions. Since the latter can be improved by an adequate choice of grid points we focused on the first aspect.

Generally, the precision of the methods reflects the precision of the variability estimation. In all presented methods variability is underestimated. Method DA, which does not incorporate information about variability at all, has the largest bias, followed by methods DAT and EDM, which assess only intra- and inter-daily variability. The other methods, which estimate variability of hourly temperatures in various ways, show smaller deviations.

The bias of the methods increases when the mean temperatures approach the grid points of the dependence function. Hence, errors are small when the temperatures in the aggregation period remain mostly in one linear part of the approximated dependence function (such as in the case study of soil respiration). Also the curvature of the dependence function influences the deviation of the expected value. If the variability is underestimated, the calculated expected value is lower than the real one for concave functions (e.g. codling moth development), and higher for convex functions (e.g. tree photosynthesis).

Most of the tested methods assume that the temperature data are normally distributed, which is not the case for the majority of the examined months. The method STOCH with normality as the only underlying assumption exhibits only a small bias, which indicates that the expected value of the dependence function is robust to departures from normality. Approximating the normal distribution by a parabola as in EDDT1 and EDDT2 underestimates slightly the expected value of the dependence function as can be seen by comparing the results of EDDT1 with EDHT1 and EDDT2 with EDHT2.

The most appropriate method for a specific model can be chosen from Figs. 4 and 5, depending on the availability of input data (cf. Table 1), on the relative importance of precision and computing efficiency, and on the required aggregation period.

When temperature data at hourly resolution are available, the dependence function can be estimated accurately with EDH, if computing time is limiting and the aggregation period is longer than about 10 days. Otherwise it can also be calculated exactly. The stochastic temperature generator STOCH is as precise as method EDH, but less efficient. If daily mean temperatures and extremes are available, EDHT1 is the best choice. If only daily extremes are obtainable and the aggregation period is longer than 3 months, EDHT2, for shorter aggregation periods the Allen method has the best performance. EDM is the only usable method if just the daily temperature means are available. Finally, if only aggregation period means of daily temperature mean and amplitude are obtainable, method DAT is the only possible choice; if only the overall temperature mean is available, the commonly used method DA has to be applied.

Method DA yields the largest mean bias of all methods, namely 8% of the exact value for the aggregation to one month and 30% for the aggregation to 10 years. Such an error can influence strongly the outcome of a model. For instance, a bias of 8% in the codling moth developmental rate with the parameter values from Lischke (1992) would correspond to 0.2 less codling moth generations per year, a bias of 30% to 0.8 less generations. This is an intolerable inaccuracy, e.g. in a pest prognosis model. The large error variability connected with this high mean error renders the method even less reliable.

For the case of highly resolved input data, the results suggest that one can optimise precision and efficiency by the appropriate choice of the aggregation method and through this by the aggregation level of the input data. This allows the use of as much information from the input data as available, provided precision and not computing time is limiting or the computing efficiency of several methods is similar, such as for EDH, EDHT1, EDHT2, Allen, and EDM. In cases of low precision or very high computing efficiency requirements and also for mostly linear dependence functions, a condensation of the input data into means and mean amplitudes and the application of method DAT or DA is reasonable. Finally, a good compromise between precision and computing efficiency requirements for high input data resolution is method EDH.

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