ELSEVIER

Ecological Modelling 98 (1997) 105–122

ECOLOGICAL MODELLING

Calculating temperature dependence over long time periods: derivation of methods

Heike Lischke *, Thomas J. Löffler, Andreas Fischlin

Systems Ecology, Institute of Terrestrial Ecology, ETH Zürich, Grabenstr. 3, CH-8952 Schlieren, Switzerland

Accepted 11 September 1996

Abstract

Rates of ecological processes are usually influenced by temperature. For simplicity and efficiency of ecosystem models it is often necessary to summarize information about temperature dependence from short, e.g. hourly, time intervals over longer, e.g. monthly, time periods, i.e. to calculate long term expected values of dependence functions. This aim can seldom be achieved by applying the temperature function to the mean temperature, because temperature dependencies are in many cases nonlinear. Therefore, we derived seven new, general methods for a temporal aggregation of temperature dependence. The methods determine the expected value interpreting either hourly temperature, daily temperature mean, or daily temperature mean and amplitude as random variables. The dependence function is approximated by a piecewise linear function. Some methods use a triangle as approximation for the daily temperature course, some a parabola as approximation for the density function of the normal distribution. The resulting methods cover a range of temperature data resolutions: monthly mean and standard deviation of hourly temperatures; daily temperature extrema; daily temperature means; and amplitudes, or daily temperature means alone. The methods can be applied to all types of dependence functions, in particular to nonlinear ones. © 1997 Elsevier Science B.V.

Keywords: Temperature dependence; Physiological time; Modelling; Aggregation; Approximations

1. Introduction

Many biologically or ecologically relevant processes are temperature dependent. Such as the growth and development of poikilothermic organisms, e.g. of insects or plants, or the processes used to synchronize

* Corresponding author. Present address: Swiss Federal Institute of Forest, Snow and Landscape Research, Zürcherstr. 111, CH-8903 Birmensdorf, Switzerland. Tel.: +41 17392533; fax: +41 17392215; e-mail: lischke@wsl.ch

PII S0304-3800(96)01907-2

an organism's life-cycle to seasonally changing environmental conditions, such as insect diapause, seed vernalization or timing of tree bud rest break. The functions $dep(T)$ by which these processes depend on temperature T are usually nonlinear, e.g. exponential or with an optimum shape. Cumulative effects of temperature on such biological processes can be measured by means of the integral

$$M(t, t_0) \underset{\text{Def.}}{:=} \int_{t_0}^{t} dep(T(\tau))\, d\tau \tag{1.0.1}$$

over a time interval (t_0, t). This integral is often referred to as 'physiological time', 'day-degree-sum', or 'heat-unit-sum'.

Temperature dependence plays also a major role in many ecological simulation models, ranging from pest prognosis models, such as Bugoff2 (Blago and Dickler, 1990) and ApfWick (Lischke and Blago, 1990; Lischke, 1992), to crop phenology models such as Biotime (Kirsta and Tarabrin, 1994), to models examining the sensitivity of ecosystems to a potential climatic change, as in the forest succession models Forska (Prentice et al., 1993), ForClim (Bugmann, 1994; Fischlin et al., 1994), and DisCForM (Lischke et al., 1996a), where physiological time determines the growth and thus, the competition of individual trees. An imprecise formulation of the temperature dependence function can seriously influence the outcome of such models, depending on the model sensitivity to the temperature dependence function. For example, a 10% error of the temperature dependence of codling moth development leads to an error of about 7 days in the simulations with a pest prognosis model (Lischke, 1992) in Central Europe. Depending on the application, such an error might not be tolerable.

The most exact approach is to calculate $M(t, t_0)$ by summing the actual values of the dependence function using temperature data in high temporal resolution, which reflect the diel or even higher frequency temperature fluctuations.

However, due to practical constraints like the lack of appropriate input data, long computation time, or the desire to keep a model as simple as possible, a larger time step is chosen in many models and temperature dependence is calculated by applying the temperature dependence function either to the mean temperatures, (e.g. monthly temperature means in ForClim) or to an interpolated temperature course (e.g. in Forska or the Biotime-model).

Yet, monthly or yearly temperature means or interpolations between means do not contain all information about the temperature variability in the studied period, particularly not about the diel variation. If the dependence function is nonlinear, which is the case for many processes, such a simple approach can lead to a loss of precision in the model outcome.

To overcome this conflict between required precision and manageability, methods are necessary to calculate physiological time as precisely as needed using as much information from the available input data as possible. The methods should be able to aggregate the temperature dependence function from the small time scale of the input data (e.g. days) to a larger time scale (e.g. years) and summarize the information about temperature dependence from short time intervals over longer periods.

Three kinds of approaches exist to deal with this problem, but rare are ideal.

(1) A possibility is to approximate the nonlinear dependence function using a linear one with a lower and an upper threshold, and to sum the daily values of this approximation as in Bugoff2 or by calculating its expected value (Aceituno, 1979). However, the use of such a linearly and monotonically increasing approximation instead of the original dependence function or a better matching nonlinear approximation as e.g. the sigmoid function proposed in (Stinner et al., 1974) or the biophysical models presented by Sharpe and DeMichele (1977) and Wagner et al. (1984) can lead to considerable loss of precision (Blago and Dickler, 1990).

(2) Other approaches apply the dependence function to an estimated daily temperature course, which has been approximated by models such as the triangulation method of Lindsey and Newman (1956), the single sine method by Baskerville and Emin (1969), the sine–sine-method of (Allen, 1976), or the sine–exponential method of Parton and Logan (1981). However, tests of some of these methods by Worner (1988) did not show a satisfactory precision for all tested sites. Moreover, if it is nonlinear, the temperature dependence function can not be evaluated in one step for each day but has to be applied to hourly values of the approximated temperature course, so that no computing time is saved compared with the use of the original hourly input data. The computational costs for solving the integral $M(t, t_0)$ over one day can only be reduced for uncomplicated, e.g. linear dependence functions as in Bugoff2.

(3) Empirical correction functions of the temperature dependence as used by Allen (1976) or Bugmann (1994) work, but confine the model application to the regions where those functions have been estimated.

Summarized, the above listed approaches are either restricted to a special, often linear type of dependence function, to a certain length of the aggregation period, or to a certain kind of input data, even if more detailed information about the temperature course in the aggregation interval is available. Or they have to be combined with empirical correction terms to yield satisfying results.

The aims of this paper are to derive several approaches for temperature dependence aggregation, which are (1) applicable for general, i.e. nonlinear temperature dependence functions; (2) for temperature input data of different resolutions; (3) which are able to use as much information as possible in the available temperature input data and (4) to work with arbitrarily large time steps, ranging from days to decades; and (5) which are generally formulated and therefore extendible to other fields of dependence functions.

2. Derivation of methods

2.1. Principles

In this section the approximation principles of the methods are described. The same general idea underlies all described approaches. If with a certain temperature x, $p_{T,\mathrm{act}}(x)$ is the relative frequency in the aggregation interval (t, t_0), e.g. 1 month, then the physiological time $M(t, t_0)$ (1.0.1) can be expressed by the integral over the dependence function of x multiplied with its absolute frequency by

$$M(t, t_0) \underset{\mathrm{Def.}}{:=} \int_{t_0}^{t} dep(T(\tau))\,\mathrm{d}\tau = (t - t_0)\int_{-\infty}^{\infty} p_{T,\mathrm{act}}(x) dep(x)\,\mathrm{d}x = (t - t_0)E[dep(T)].$$

Thereby $E[dep(T)]$ is the expected value of the temperature dependence function $dep(T)$. The problem is to find a reliable estimator for $E[dep(T)]$ in (t_0, t), given the mean value and standard deviation or only the mean value of temperature or related variables, e.g. temperature extrema.

Here we derive seven methods for the estimation of $E[dep(T)]$. The symbols are explained in Table 1. For the mnenomic abbreviations of the methods see Table 2. The approximations used for the estimation and the exact algorithms are given in Section 2.2 and in the Appendix A, respectively. For sake of simplicity we consider the aggregation from an hourly to a monthly time interval, but the methods can also be applied for other aggregations from all time intervals of less than 1 day to larger ones, e.g. 1 year or decade.

2.1.1. Methods using the hourly temperature as random variable

We describe two approaches, abbreviated as DA and EDH, respectively, which regard the hourly temperature as a normally distributed random variable with the density function $p_T(x)$. In the widely used approach DA, the expected value is approximated by applying the dependence function directly to the mean temperature value μ_T in the regarded period, i.e.

$$E[dep(T)] \simeq dep(\mu_T) \qquad (2.1.1)$$

In approach EDH the expected value $E[dep(T)]$ of the hourly values of the temperature dependence is calculated explicitly by

$$T \sim N(\mu_T, \sigma_T) \Rightarrow p_T(x) = \frac{\exp[-(x-\mu_T)^2/2\sigma_T^2]}{\sigma_T\sqrt{2\pi}} \qquad (2.1.2)$$

$$E[dep(T)] = \int_{-\infty}^{\infty} dep(x) p_T(x)\, dx \qquad (2.1.3)$$

2.1.2. Methods using daily temperature mean, amplitude, and extrema as random variables

If there is no hourly input data, but data about the daily temperature means, amplitudes, or extrema available, then the following six methods EDHT1, EDHT2, EDM, DAT, EDDT1 and EDDT2 can be used.

Methods approximating the statistical parameters of hourly temperatures.

(1) Approach EDHT1 applies the same algorithm as EDH, but estimates the mean μ_T and standard deviation σ_T (Eq. (A.2.3)) of the hourly temperatures from the means $\mu_{\bar{T}}$, μ_Δ and standard deviations $\sigma_{\bar{T}}$, σ_Δ of the daily mean $\bar{T}$ and amplitude Δ.

(2) Approach EDHT2 corresponds to EDHT1, with the difference of calculating the daily temperature mean $\bar{T}$ and amplitude Δ from the daily extrema by $\bar{T} \simeq \bar{T}_m = (T_{min} + T_{max})/2$ and $\Delta = T_{max} - T_{min}$.

Table 1
Symbols

Symbol	Meaning	Unit
t	Time	Days
$T(t)$	Temperature at time t	°C
$\tilde{T}(t)$	Approximation of temperature at time t	°C
$\bar{T}$	Daily temperature mean	°C
T_{min}	Daily minimum temperature	°C
T_{max}	Daily maximum temperature	°C
t_{max}	Time of daily maximum temperature	Day
Δ	Daily temperature amplitude	°C
$dep(T)$	Temperature dependence function	—
$d\tilde{e}p(T)$	Approximation of temperature dependence function	—
$d\tilde{e}p_i(T)$	ith linear part of approximation for temperature dependence function	—
d_i	Grid point for discretization of $d\tilde{e}p(T)$, lower temperature threshold of $d\tilde{e}p_i(T)$	°C
d_{i+1}	Upper temperature threshold of $d\tilde{e}p_i(T)$	°C
$E[X]$	Expected value of random variable X	Same as X
p_X	Probability density function of random variable X	—
$\tilde{p}_X$	Approximation of probability density function	—
μ_X	Mean of random variable X	Same as X
σ_X	Standard deviation of random variable X	Same as X
ζ_n, ρ_m	Coefficients of $\tilde{p}_{\bar{T}}(y)$ and $\tilde{p}_\Delta(z)$ in polynomial form	
$\vdash$, $\dashv$	Boundaries of interval where $\tilde{p}_X(x) \neq 0$, $X = \bar{T}$, Δ	°C
DEP	Daily temperature dependence function integral	—
$D\tilde{E}P_{i,v}$, $(k_1, k_2, \bar{T}, \Delta)$	Daily integral over ith linear part of approximation of temperature dependence function	—
$\xi_{j,l}$ $(k_1, k_2) \in C_D$	Coefficients of $D\tilde{E}P_{i,v}$, $(k_1, k_2, \bar{T}, \Delta)$ in polynomial form	—

Table 2
Overview over temperature dependence aggregation methods

Method	Abbr.	Input data			Algorithm		
		Time resolution	Variables	Statistical parameters	Exact formula	Approximation	Approx. formula
Expected value dependence function of hourly temperatures	EDH	Hours	T	μ_T, σ_Δ	2.1.3	dep	A.1.2
Expected value of dependence function of hourly temperatures approx. by triangle based on mean and amplitude	EDHT1	Days	$\bar{T}$, $\Delta = T_{max} - T_{min}$	$\mu_{\bar{T}}$, $\sigma_{\bar{T}}$, μ_Δ, σ_Δ	2.1.3	dep; TC	A.1.2, A.2.3
Expected value of dependence function of hourly temperature approx. by triangle based on mean and amplitude	EDDT1	Days	$\bar{T}$, $\Delta = T_{max} - T_{min}$	$\mu_{\bar{T}}$, $\sigma_{\bar{T}}$, μ_Δ, σ_Δ	2.1.7	dep, TC,	A.3.12
Expected value of dependence function of hourly temperatures approx. by triangle based on extrema	EDHT2	Days	$\bar{T}_m = (T_{max} + T_{min})/2$ $\Delta = T_{max} - T_{min}$	$\mu_{\bar{T}_m}$, $\sigma_{\bar{T}_m}$ μ_Δ, σ_Δ	2.1.3	dep, TC	A.1.2, A.2.3
Expected value of dependence function of daily temperature triangle based on extrema	EDDT2	Days	$\bar{T}_m = (T_{max} + T_{min})/2$ $\Delta = T_{max} - T_{min}$	$\mu_{\bar{T}_m}$, $\sigma_{\bar{T}_m}$ μ_Δ, σ_Δ	2.1.7	dep; TC; ND	A.3.12
Expected value of dependence function of daily temperature mean	EDM	Days	$\bar{T}$	$\mu_{\bar{T}}$, σ_Δ	2.1.4	dep	A.1.2
Dependence function of average daily temperature triangle	DAT	Months	$\mu_{\bar{T}}$, μ_Δ	$\mu_{\bar{T}}$, μ_Δ	2.1.6	dep; TC	A.3.9
Dependence function of average temperature	DA	Months	$\mu_T = \mu_T$	$\mu_{\bar{T}}$	2.1.1	dep	2.1.1

Expected are divided according to the type of method (explicit Methods value calculation or dependence function of average input), the resolution and kind of the needed input data: T, hourly temperature; T_{max}, T_{min}, daily temperature extrema; Δ, daily temperature amplitude; $\bar{T}$, daily temperature mean; $\bar{T}_m$, approximated daily temperature mean; $\mu_{\bar{T}}$, monthly mean temperature; and σ, monthly mean amplitude, the statistical parameters estimated from these data: μ, mean; and σ, standard deviation, and the approximations used: dep, dependence function; TC, daily temperature course; and ND, normal distribution.
References to the formulae are given in columns 'exact formula' and 'approx. formula'. Method DA is a commonly used approach.

(3) In approach EDM the expected value is calculated explicitly as in EDH, but with the daily mean temperature $\bar{T}$ as random variable, which corresponds to the assumption that hourly and daily mean temperatures have a similar variance.

$$\bar{T} \sim N(\mu_{\bar{T}}, \sigma_{\bar{T}}) \Rightarrow p_{\bar{T}}(y) = \frac{\exp[-(y-\mu_{\bar{T}})^2/2\sigma_{\bar{T}}^2]}{\sigma_{\bar{T}}\sqrt{2\pi}}$$

$$E[dep(T)] = \int_{-\infty}^{\infty} dep(y) p_{\bar{T}}(y)\, \mathrm{d}y. \tag{2.1.4}$$

Methods approximating the daily temperature dependence function. In a first step the temperature course $T(t)$ for each day is approximated by a function $\tilde{T}(t, \bar{T}, \Delta)$ (Eq. (2.2.2)) of time, daily average temperature $\bar{T}$, and daily temperature amplitude Δ. Then the daily integral $DEP(\bar{T}, \Delta)$ of the temperature dependence function $dep(T)$ applied to this approximated temperature course $\tilde{T}(t, \bar{T}, \Delta)$ is evaluated (Eqs. (A.3.6) and (A.3.7)) by

$$DEP(\bar{T}, \Delta) \underset{\text{Def.}}{:=} \int_0^1 dep(\tilde{T}(\tau, \bar{T}, \Delta))\, \mathrm{d}\tau \simeq \int_0^1 dep(T(\tau))\, \mathrm{d}\tau \tag{2.1.5}$$

for each day normalized to the interval (0, 1) of the period (t_0, t). In a second step the expected value $E[DEP(\bar{T}, \Delta)]$ of $DEP(\bar{T}, \Delta)$ for all days in (t_0, t) is determined.

(1) Approach DAT approximates $E[DEP(\bar{T}, \Delta))$ by applying DEP to the average daily temperature course, which is characterized by the average daily temperature mean $\mu_{\bar{T}}$ and the average daily temperature amplitude μ_Δ, i.e.

$$E[DEP(\bar{T}, \Delta)] \simeq DEP(\mu_{\bar{T}}, \mu_\Delta). \tag{2.1.6}$$

(2) In approach EDDT1 the expected value of $DEP(\bar{T}, \Delta)$ is calculated, regarding daily temperature mean $\bar{T}$ and amplitude Δ as independently normally distributed random variables with means $\mu_{\bar{T}}$ and μ_Δ, standard deviations $\sigma_{\bar{T}}$ and σ_Δ, and density functions $p_{\bar{T}}(y)$ and $p_\Delta(z)$, which are defined analogously to Eq. (2.1.2). The expected value $E[DEP(\bar{T}, \Delta)]$ is defined by

$$E[DEP(\bar{T}, \Delta)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} DEP(y, z) p_{\bar{T}}(y)\, \mathrm{d}y\, p_\Delta(z)\, \mathrm{d}z. \tag{2.1.7}$$

(3) Approach EDDT2 corresponds to EDDT1, except that the daily temperature mean $\bar{T}$ is calculated by $\bar{T} \simeq \bar{T}_m = (T_{\min} + T_{\max})/2$.

2.2. Approximations

In this section the approximations underlying all derived methods are described. The integrals in Eqs. (2.1.3), (2.1.4), and (2.1.7) are of the type $\int e^{-(x-a)^2} f(x)\, \mathrm{d}x$. These integrals can be solved explicitly only, for specific, comparably simple dependence functions dep, such as in the following example.

Example 2.1. The solution of (Eq. (2.1.3)) for the exponential temperature dependence function $dep_{Q_{10}}(T) = e^{\alpha T}$, $\alpha = \mathrm{Ln}(Q_{10})/10$, where Q_{10} is the ratio between dependence function values at temperatures $T + 10°\mathrm{C}$ and T, can be directly evaluated from Eq. (2.1.3) as

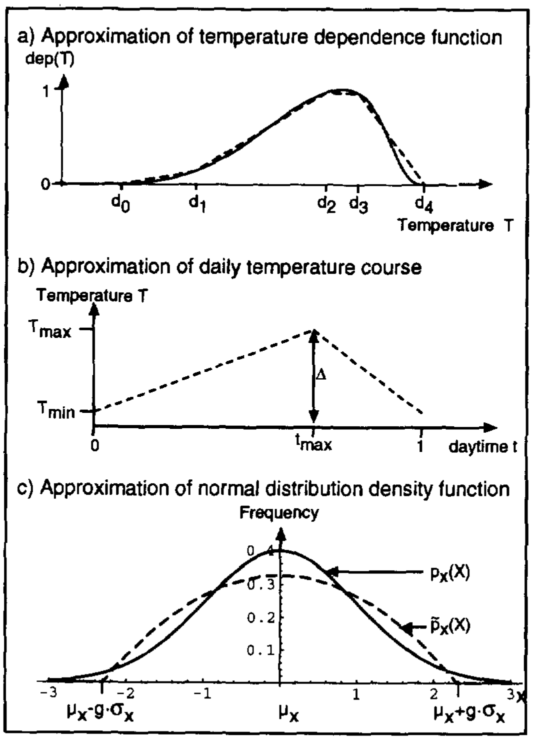

Fig. 1. Approximations used in the aggregation methods: (a) approximation of the temperature dependence function by a piecewise linear function, defined by a set of grid points d_i and the corresponding values of the dependence function; (b) approximation of the daily temperature course using a triangle between $T_{\min}$ and $T_{\max}$. The maximum temperature is reached at time $t_{\max}$; and (c) approximation of the density function $p_X(x)$ of the normal distribution by the parabola $\tilde{p}_X(x)$, $X = \bar{T}$, Δ (μ_X and σ_X are mean and standard deviation of X).

$$E[\mathrm{dep}_{Q_{10}}(T)] = \int_{-\infty}^{\infty} \mathrm{dep}_{Q_{10}}(x) p_T(x)\, \mathrm{d}x = \frac{1}{\sigma_T\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\alpha x} \exp[-(x-\mu_T)^2/2\sigma_T^2]\, \mathrm{d}x$$

$$= \frac{1}{\sigma_T\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp[(2\alpha\sigma_T^2 x - x^2 + 2x\mu_T - \mu_T^2)/2\sigma_T^2]\, \mathrm{d}x$$

$$= \frac{1}{\sigma_T\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp[(-x^2 + 2x(\mu_T + \alpha\sigma_T^2) - (\mu_T + \alpha\sigma_T^2)^2 + (\mu_T + \alpha\sigma_T^2)^2 - \mu_T^2)/2\sigma_T^2]\, \mathrm{d}x$$

$$= \exp[((\mu_T + \alpha\sigma_T^2)^2 - \mu_T^2)/2\sigma_T^2] \underbrace{\left\{\frac{1}{\sigma_T\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp[-(x - (\mu_T + \alpha\sigma_T^2))^2/2\sigma_T^2]\, \mathrm{d}x\right\}}_{=1}$$

$$= \exp[\alpha\mu_T + 0.5\alpha^2\sigma_T^2].$$

To avoid restriction to specific dependence function types and to obtain solutions for any function *dep*, we introduce the approximations (Fig. 1(a–c)):

- The temperature dependence function $dep(T)$ is approximated by a piecewise linear function $\tilde{dep}(T)$, which consists of the linear pieces $\tilde{dep}_i(T)$, $i = 0, \ldots, n_d - 1$ defined by the n_d grid points d_i.

$$\tilde{dep}_i(T) = \begin{cases} dep(d_i) + (T - d_i)\dfrac{dep(d_i + 1) - dep(d_i)}{d_{i+1} - d_i}, & d_i \leq T < d_{i+1} \\ 0, & \text{else} \end{cases} \quad (2.2.1)$$

In Fig. 1a an approximation with four linear parts is shown. The optimal grid points can be obtained by minimizing numerically the area between $dep(T)$ and $\tilde{dep}(T)$.

- The daily temperature course is approximated by an asymmetric triangle $\tilde{T}(t)$ (Fig. 1b) with the same minimum temperature at the beginning and end of the day and a variable time point t_{max} of the maximum temperature.

$$T_{min} = \bar{T} - \frac{\Delta}{2}; \quad T_{max} = \bar{T} + \frac{\Delta}{2};$$

$$\tilde{T}(t) = \begin{cases} \bar{T} - \dfrac{\Delta}{2} + t\dfrac{\Delta}{t_{max}}, & 0 \leq t < t_{max} \\ \bar{T} - \dfrac{\Delta}{2} + (1 - t)\dfrac{\Delta}{1 - t_{max}}, & t_{max} \leq t < 1 \end{cases} \quad (2.2.2)$$

- The normal distribution density functions $p_{\bar{T}}(y)$ and $p_{\Delta}(z)$ in Eq. (2.1.7) are each approximated by a polynomial $\tilde{p}$ of second order which is set to zero for values x, $y < 0$ (in Fig. 1c: $x < \mu_X - g\sigma_X$ and $x > \mu_X + g\sigma_X$, $X = \bar{T}$, Δ). The parameter g determines the width and height of the parabola. In Fig. 1c the approximation for $g = 2.3$ is shown.

$$\tilde{p}_X(y) = \begin{cases} \dfrac{3}{g^3\sigma_X}\left(\left(\dfrac{g}{2}\right)^2 - \left(\dfrac{y - \mu_X}{2\sigma_X}\right)^2\right), & \mu_X - g\sigma_X < y < \mu_X + g\sigma_X \\ 0, & \text{, else} \end{cases} \quad (2.2.3)$$

with $X = \bar{T}$, Δ.

With these approximations we get (piecewise) polynomials for all functions in the integrals (Eq. (2.1.3)), (Eq. (2.1.4)), and (Eq. (2.1.7)). The so replaced integrals can be solved analytically with the help of symbolic calculation software as, e.g. Mathematica or Maple.

2.3. Algorithms and implementation

The algorithms of the new approaches (EDH, EDM, EDHT1, EDHT2, DAT, EDDT1, and EDDT2), which are based on the principles of Section 2.1 and the approximations of Section 2.2, are derived in detail in the appendix. Table 2 gives an overview of the methods, their temporal resolution, input data needs, cross references to the equations and any approximations used. The methods have been implemented on SUN and Macintosh in Modula-2. The source codes can be obtained by anonymous ftp from:

ftp.ito.umnw.ethz.ch/pub/mac/EofTempDep

3. Discussion

In this paper a range of new approaches for aggregating temperature dependence functions to longer time periods have been derived. The methods are constructed for a variety of input data resolutions and allow the inclusion of temporal temperature variability in ecological models. Thus, an appropriate method now can be chosen from this set (Table 2), depending on the available input data, the needed aggregation period, and the necessary precision. The main characteristics and differences of the methods are:

(1) Method EDH takes into account the intra daily variability by using hourly input data and hence including the temperature variance.

(2) Methods EDDT1, EDHT1, EDDT2, EDHT2, and DAT extract the information about the intra daily variability from daily temperature amplitudes by assuming a triangle-shaped temperature course, which is either used to estimate the statistical parameters of the hourly temperatures or to calculate the daily dependence function and its expected value.

(3) Method EDM uses the inter-daily variability by the variance of daily mean temperatures, but neglects the intra daily variability.

Thus, all of the presented approaches are able, to different extents, to include temporal temperature variability, in contrast to the widely used application of the dependence function to (long term) temperature means (approach DA in Table 2).

However, all methods have a bias arising from the used approximations and assumptions. They assume the temperature variables to be normally distributed and temperature mean and amplitude to be independent of each other, which is probably not always correct. The assumption of a daily triangle temperature course similar to the triangulation method of Lindsey and Newman (1956), might also appear crude. However, physiological time calculated with this triangulation can be sufficiently precise, if the times of the daily temperature maxima are known, as shown for the example of codling moth development (Lischke, 1991). Thus, for an adequate use of the triangle approximation either the temperature maximum time is required for each day or a method which calculates the daily dependence function based on the triangulation independently of this time.

The latter holds for the methods EDDT1 and EDDT2, where the temperature maximum time drops out during the calculation of the daily temperature dependence. This could be an advantage over the methods EDHT1 and EDHT2 and also over the sine–sine-method of Allen (1976), because in their daily dependence approximation this time still appears, and hence has to be determined or estimated, e.g. to be at noon.

To assess the effects of the aforementioned potential biases and the applicability of the presented methods, the precision and efficiency of the methods have been tested (Lischke et al., 1996b) in several ecological applications and compared with other common methods. The tests revealed that it can be crucial to use all available variability information dependent on the precision requirements to obtain satisfying results. Also, the approaches EDH, EDHT1, and EDHT2 combined high precision with high speed at their respective levels of resolution. The effect of the bias introduced by assuming the temperature maximum to occur at noon in EDHT1 and EDHT2 turned out to be negligible.

The presented methods can be used in a wide range of ecological models where variable abiotic factors are affecting the dynamics, e.g. in pest prognosis models. They can be particularly useful where dynamics which still depend on smaller time scale variations have to be simulated on large time scales, as, e.g. weather dependent plant growth in dynamic vegetation models which are used to assess the impact of climate change over centuries. For instance, the forest succession model ForClim reacts very sensitively (Fischlin et al., 1994) to whether the climate input is formulated as constant input or by a stochastic weather generator on the monthly scale but runs for several hundred years. Another example are models for the simulation of the forest carbon cycle as reviewed by Perruchoud and Fischlin (1995), which depend on temperature and run for even longer simulation periods (e.g. 15 000 years).

The construction of the approaches is probably not restricted to the specific approximations we presented here, other ones could be chosen as, e.g. quadratic polynomials for the daily temperature course, exponential functions to approximate the temperature dependence function (as shown in example 2.1), or piecewise linear polynomials to approximate density functions. The latter could extend the range of applicability beyond normal distributions, even to empirical ones.

The approaches are also not restricted to dependence functions of temperature. The methods EDH and EDM which do not assume a certain daily temperature course could also be applied to dependence functions of other abiotic factors, or more generally to the calculation of arbitrary functions of normally distributed random variables. We used, e.g. the method EDH successfully to calculate the expected values of a nonlinear light dependence function in the forest dynamics model DisCForm (Lischke et al., 1996a).

The concept of approximating the daily temperature course, which is the basis of the methods DAT, EDHT1, EDHT2, EDDT1, and EDDT2 could be transferred to other periodicities, as e.g. inter-decadal temperature oscillations (Mann et al., 1995) or the yearly temperature course.

This would allow the estimation of long term dependence functions of monthly temperature means, given yearly statistic parameters of extrema and means of daily or monthly temperature means.

The methods are even not restricted to temporal variability. It may also be possible to apply them to spatially varying input variables, e.g. during an spatial model upscaling.

4. Conclusions

Now we have a variety of methods at our disposal, which can be applied to every temperature dependence function by simple linearization. They are suitable for different temperature input data resolutions, e.g. minutely or hourly temperature, daily mean and daily amplitude, daily extrema, monthly mean and monthly mean day-amplitude and monthly mean. With these methods it is possible to use all the information about the variability in the input data as available through daily amplitudes or standard deviations of hourly temperatures, and they can be used for arbitrarily large time steps ranging from days to millenia. Finally they can be applied to any kind of dependence function in many fields of ecological modelling applications.

Acknowledgements

This work has been supported by the Swiss Federal Institute of Technology (ETH) Zurich and by the Swiss National Science Foundation, grants no.5001-35172 and 31-31142.91. Thanks to A. McLellan, J. Shykoff, and B. Roy for checking the English of the manuscript.

Appendix A. Algorithms

Here we derive in detail the algorithms, which are used to evaluate the new approaches (EDH, EDM, EDHT1, EDHT2, DAT, EDDT1, and EDDT2) the principles of which were presented in Section 2.1. Approximated dependence, daily temperature course or distribution density functions are marked by a tilde.

The seven new methods approximate the temperature dependence function $dep(T)$ by $d\tilde{e}p(T)$ (Eq. (2.2.1)). Thus, this piecewise linear function $d\tilde{e}p(T)$ has to be defined suitably by the grid points d_i, $i = 0, \ldots, n_d - 1$, e.g. by numerically minimizing the square distances between $dep(T)$ and $d\tilde{e}p(T)$. Then the expected value of $d\tilde{e}p(T)$ can be treated as a sum of the expected values of the different linear pieces, i.e. $E[d\tilde{e}p(T)] = \Sigma_{i=1}^{n_d} E[d\tilde{e}p_i(T))$. In the following it is therefore sufficient to explain the evaluation of $E[d\tilde{e}p_i(T)]$.

A.1. Approach EDH

For approach EDH we substitute in Eq. (2.1.3) the temperature dependence function $dep(T)$ with the approximation $d\tilde{e}p_i(T)$ (Eq. (2.2.1)). In this way we get the approximated expected value $E[d\tilde{e}p_i(T)]$ of each ith part of the dependence function as

$$E[d\tilde{e}p_i(T)] = \int_{d_i}^{d_{i+1}} d\tilde{e}p_i(x) \frac{\exp[-(x-\mu_T)^2/2\sigma_T^2]}{\sigma_T\sqrt{2\pi}} \, dx$$

$$= \int_{d_i}^{d_{i+1}} \left(dep(d_i) + (x-d_i)\frac{dep(d_{i+1})-dep(d_i)}{d_{i+1}-d_i} \right) \frac{\exp[-(x-\mu_T)^2/2\sigma_T^2]}{\sigma_T\sqrt{2\pi}} \, dx$$

$$= \int_{(d_i-\mu_T)/\sqrt{2}\sigma_T}^{(d_{i+1}-\mu_T)/\sqrt{2}\sigma_T} \left(dep(d_i) + (y\sqrt{2}\sigma_T+\mu_T-d_i)\frac{dep(d_{i+1})-dep(d_i)}{d_{i+1}-d_i} \right) \frac{\exp[-y^2]}{\sigma_T\sqrt{2\pi}} \sqrt{2}\sigma_T \, dy$$

$$= \int_{(d_i-\mu_T)/\gamma}^{(d_{i+1}-\mu_T)/\gamma} \beta_i \exp[-y^2] \, dy + \int_{(d_i-\mu_T)/\gamma}^{(d_{i+1}-\mu_T)/\gamma} \alpha_i y \exp[-y^2] \, dy \qquad \text{(A.1.1)}$$

with $y=(x-\mu_T)/\sqrt{2}\sigma_T$, $\alpha_i=(\gamma/\sqrt{\pi})(dep(d_{i+1})-dep(d_i))/(d_{i+1}-d_i)$, $\beta_i=(dep(d_i)/\sqrt{\pi})+(\mu_T-d_i)dep(d_{i+1})-dep(d_i))/((d_{i+1}-d_i)\sqrt{\pi})$, and $\gamma=\sqrt{2}\sigma_T$. The second integral can be solved directly, the first one yields the error function $\text{Erf}(x)=(2/\sqrt{\pi})\int_0^x \exp[-t^2] \, dt$, whose values can be obtained from tabulations or its series expansion $\Sigma_{\kappa=1}^{\kappa_{\max}} (x^{2\kappa-1}(-1)^{\kappa-1})/((\kappa-1)!(2\kappa-1))$.

The solution is

$$E[d\tilde{e}p_i(T)] = \left(\beta_i \frac{\sqrt{\pi}}{2}\text{Erf}(y) - \frac{\alpha_i}{2}\exp[-y^2] \right) \Bigg|_{(d_i-\mu_T)/\gamma}^{(d_{i+1}-\mu_T)/\gamma}$$

$$= \beta_i \frac{\sqrt{\pi}}{2}\left(\text{Erf}\left(\frac{d_{i+1}-\mu_T}{\gamma}\right) - \text{Erf}\left(\frac{d_i-\mu_T}{\gamma}\right) \right)$$

$$-\frac{\alpha_i}{2}(\exp[-((d_{i+1}-\mu_T)/\gamma)^2] - \exp[-((d_i-\mu_T)/\gamma)^2]). \qquad \text{(A.1.2)}$$

A.2. Approaches EDM, EDHT1, and EDHT2

The approaches EDM, EDHT1, and EDHT2 are based on approach EDH (Eq. (A.1.2)) but differ in the way they estimate the data T, μ_T, and σ_T

(1) For approach EDM in Eq. (A.1.2) T, μ_T, and σ_T are replaced by $\bar{T}$, $\mu_{\bar{T}}$ and $\sigma_{\bar{T}}$.

(2) In approach EDHT2 in a first step the daily mean temperatures are approximated by $\bar{T} \simeq \bar{T}_m = (T_{\min}+T_{\max})/2$ for each day. Hence we can approximate $\mu_{\bar{T}} \simeq \mu_{\bar{T}_m}$ and $\sigma_{\bar{T}} \simeq \sigma_{\bar{T}_m}$.

(3) Then in both approaches EDHT1 and EDHT2 we derive the mean μ_T and variance σ_T of the hourly temperatures from the mean $\mu_{\bar{T}}$ and μ_Δ and the variance $\sigma_{\bar{T}}$ and σ_Δ of the daily temperature means and amplitudes.

The approximation of μ_T is easy, since $\mu_T = \mu_{\bar{T}}$.

To obtain the approximation $\tilde{\sigma}_T$ of σ_T we assume that the temperature course $\tilde{T}_\varsigma(t)$ at a specific day ς follows a triangle, analogously to the approximation in Fig. 1b and Eq. (2.2.2). The triangle is symmetric with maximum at noon and equal minima at the beginning and end of the day. Hence we get

$$\tilde{T}_\varsigma(t) = \begin{cases} \bar{T}_\varsigma - \dfrac{\Delta_\varsigma}{2} + t\dfrac{\Delta_\varsigma}{0.5}, & 0 \le t < 0.5 \\ \bar{T}_\varsigma - \dfrac{\Delta_\varsigma}{2} + (1-t)\dfrac{\Delta_\varsigma}{0.5}, & 0.5 \le t < 1 \end{cases}$$

The variance σ_T of this approximated temperature course $\tilde{T}(t)$ during m days is given through the mean quadratic distance of each temperature value to the mean temperature by

$$\begin{aligned}
\tilde{\sigma}_T^2 &= \frac{1}{m}\sum_{\varsigma=1}^{m}\int_0^1 (\tilde{T}_s(t)-\mu_T)^2\,\mathrm{d}t = \frac{2}{m}\sum_{\varsigma=1}^{m}\int_0^{0.5}\left(\left\{\bar{T}_\varsigma - \frac{\Delta_\varsigma}{2} - \mu_T\right\}_\alpha + t\{2\Delta_\varsigma\}_\beta\right)^2 \mathrm{d}t \\
&= \frac{2}{m}\sum_{\varsigma=1}^{m}\int_0^{0.5}(\alpha^2 + 2\alpha\beta t + \beta^2 t^2)\,\mathrm{d}t = \frac{1}{m}\sum_{\varsigma=1}^{m}\left(\alpha^2 + \frac{\alpha\beta}{2} + \frac{\beta^2}{12}\right) \\
&= \frac{1}{m}\sum_{\varsigma=1}^{m}(\bar{T}_\varsigma - \mu_T)^2 - \Delta_\varsigma(\bar{T}_\varsigma - \mu_T) + \frac{\Delta_\varsigma^2}{4} + \left(\bar{T}_\varsigma - \mu_T - \frac{\Delta_\varsigma}{2}\right)\Delta_\varsigma + \frac{4\Delta_\varsigma^2}{12} \\
&= \left\{\frac{1}{m}\sum_{\varsigma=1}^{m}(\bar{T}_\varsigma - \mu_T)^2\right\}_{\sigma_{\bar{T}}^2} + \frac{1}{m}\sum_{\varsigma=1}^{m}\left(\{\Delta_\varsigma(-\bar{T}_\varsigma + \bar{T}_\varsigma - \mu_T + \mu_T)\}_0 + \Delta_\varsigma^2\left(\frac{1}{4} - \frac{1}{2} + \frac{1}{3}\right)\right) \\
&= \sigma_{\bar{T}}^2 + \frac{1}{12}\frac{1}{m}\sum_{\varsigma=1}^{m}\Delta_\varsigma^2 = \sigma_{\bar{T}}^2 + \frac{1}{12}\frac{1}{m}\sum_{\varsigma=1}^{m}(\Delta_\varsigma^2 - 2\Delta_\varsigma\mu_\Delta + \mu_\Delta^2 + 2\Delta_\varsigma\mu_\Delta - \mu_\Delta^2) \\
&= \sigma_{\bar{T}}^2 + \frac{1}{12}\left\{\frac{1}{m}\sum_{\varsigma=1}^{m}(\Delta_\varsigma - \mu_\Delta)^2\right\}_{\sigma_\Delta^2} + \frac{1}{12}\frac{1}{m}\sum_{\varsigma=1}^{m}(2\Delta_\varsigma\mu_\Delta - \mu_\Delta^2) = \sigma_{\bar{T}}^2 + \frac{1}{12}\sigma_\Delta^2 + \frac{2\mu_\Delta}{12m}\sum_{\varsigma=1}^{m}(\Delta_\varsigma) - \frac{1}{12}\mu_\Delta^2 \\
&= \sigma_{\bar{T}}^2 + \frac{1}{12}\sigma_\Delta^2 + \frac{2\mu_\Delta}{12}\mu_\Delta - \frac{1}{12}\mu_\Delta^2 = \sigma_{\bar{T}}^2 + \frac{1}{12}\sigma_\Delta^2 + \frac{1}{12}\mu_\Delta^2.
\end{aligned}$$

This gives the result

$$\Rightarrow \tilde{\sigma}_T = \sqrt{\frac{1}{12}(\sigma_\Delta^2 + \mu_\Delta^2) + \sigma_{\bar{T}}^2}. \tag{A.2.3}$$

Then the algorithm EDH (Eq. (A.1.2)) is used with the obtained μ_T and $\tilde{\sigma}_T$

A.3. Approaches DAT, EDDT1, and EDDT2

For the following three methods first the daily value of the ith part of the dependence function is calculated analytically. Then the expected value of this daily value is approximated.

Daily dependence function integral. The approximated daily integral $DEP(\bar{T}, \Delta)$ (Eq. (2.1.5)) over the approximated dependence function part $d\tilde{e}p_i(T)$ (Eq. (2.2.1)) applied to the approximated temperature course $\tilde{T}(t)$ (Eq. (2.2.2)) (illustrated by Fig. 2) is given by

$$\begin{aligned}
D\tilde{E}P_i &= \int_0^1 d\tilde{e}p_i(\tilde{T}(\tau))\,\mathrm{d}\tau = \int_0^1 dep(d_i) + (\tilde{T}(\tau) - d_i)\left\{\frac{dep(d_{i+1}) - dep(d_i)}{d_{i+1} - d_i}\right\}_{\alpha_i}\mathrm{d}\tau \\
&= \int_{\rfloor i,1}^{\rceil i,1}\left\{dep(d_i) + \alpha_i\left(\bar{T} - \frac{\Delta}{2} - d_i\right)\right\}_{\beta_i} + \alpha_i\tau\frac{\Delta}{t_{\max}}\,\mathrm{d}\tau + \int_{\rfloor i,2}^{\rceil i,2}\left\{dep(d_i) + \alpha_i\left(\bar{T} - \frac{\Delta}{2} - d_i\right)\right\}_{\beta_i}
\end{aligned}$$

$$+\alpha_i(1-\tau)\frac{\Delta}{1-t_{max}}d\tau=\beta_i(\rceil_{i,1}-\rfloor_{i,1})+\frac{\alpha_i\Delta}{2t_{max}}(\rceil^2_{i,1}-\rfloor^2_{i,1})+\beta_i(\rceil_{i,2}-\rfloor_{i,2})$$

$$+\frac{\alpha_i\Delta}{1-t_{max}}\left(\rceil_{i,2}-\rfloor_{i,2}-\frac{1}{2}(\rceil^2_{i,2}-\rfloor^2_{i,2})\right)$$

$$=\beta_i\{(\rceil_{i,1}-\rfloor_{i,1}+\rceil_{i,2}-\rfloor_{i,2})\}_{f_{\beta_i}}$$

$$+\alpha_i\Delta\left\{\frac{(1-t_{max})(\rceil^2_{i,1}-\rfloor^2_{i,1})-t_{max}(\rceil^2_{i,2}-\rfloor^2_{i,2})+2t_{max}(\rceil_{i,2}-\rfloor_{i,2})}{2t_{max}(1-t_{max})}\right\}_{f_{\alpha_i}}=\beta_i f_{\beta_i}+\alpha_i\Delta f_{\alpha_i} \quad \text{(A.3.4)}$$

with $\rfloor_{i,1}=\max(0,\ t_{i,1})$, $\rceil_{i,1}=\min(t_{i+1,1},\ t_{max})$, $\rfloor_{i,2}=\max(t_{max},\ t_{i+1,2})$, and $\rceil_{i,2}=\min(t_{i,2},\ 1)$.

Fig. 2 shows that $t_{i,1}$, $t_{i+1,1}$, $t_{i,2}$, and $t_{i+1,2}$ are the times when the temperature reaches d_i and d_{i+1}, the lower and upper threshold of the dependence function during the increasing respectively decreasing part of the approximated daily time course $\tilde{T}(t)$. These times can be calculated by the inverse function of Eq. (2.2.2), i.e.

$$t=\begin{cases} t_{max}\dfrac{\tilde{T}-\bar{T}+\Delta/2}{\Delta}, & 0\le t<t_{max}\\ 1-(1-t_{max})\dfrac{\tilde{T}-\bar{T}+\Delta/2}{\Delta}, & t_{max}\le t<1\end{cases} \quad \text{(A.3.5)}$$

The integration borders $\rceil$ and $\rfloor$ depend on T_{min} and T_{max} as well as on the thresholds d_i and d_{i+1}. According to whether the temperatures of the regarded day remain between these thresholds, cut them or lie outside of them, we get four different cases of (T_{min}, d_i) and (T_{max}, d_{i+1}) combinations, where $d\tilde{e}p\neq 0$ (Fig. 2), mentioned in (Allen, 1976). Because $T_{min}=\bar{T}-\Delta/2$ and $T_{max}=\bar{T}+\Delta/d$, these four cases correspond to four combinations of $\bar{T}$ and Δ drawn as shaded areas Ω_v, $v=1, \ldots, 4$ in Fig. 3a. The values of the integral boundaries $\rfloor_{i,1}$, $\rceil_{i,1}$, $\rfloor_{i,2}$, and $\rceil_{i,2}$ are given in Table 3. The resulting values for f_{α_i} and f_{β_i} in Eq. (A.3.4) for the four cases are listed in Table 4.

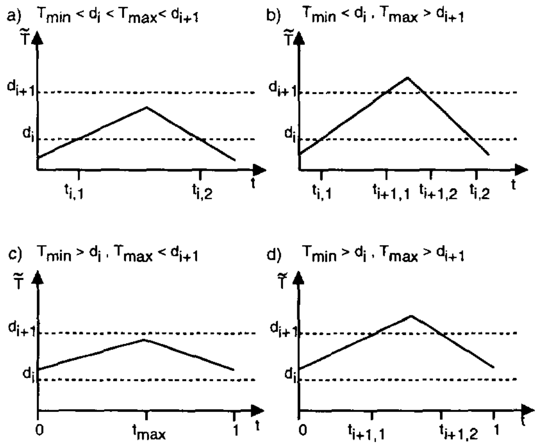

Fig. 2. Integration intervals of the ith part of the daily temperature dependence integral. Four different cases of daily temperature triangles, defined through the position of the temperature extrema relative to the threshold values d_i and d_{i+1} of the dependence function approximation. The temperature approximation reaches the thresholds d_i resp. d_{i+1} at times $t_{i,1}$ resp. $t_{i+1,1}$ in the increasing part of the triangle, and at the times $t_{i,2}$ resp. $t_{i+1,2}$ in the decreasing part of the triangle.

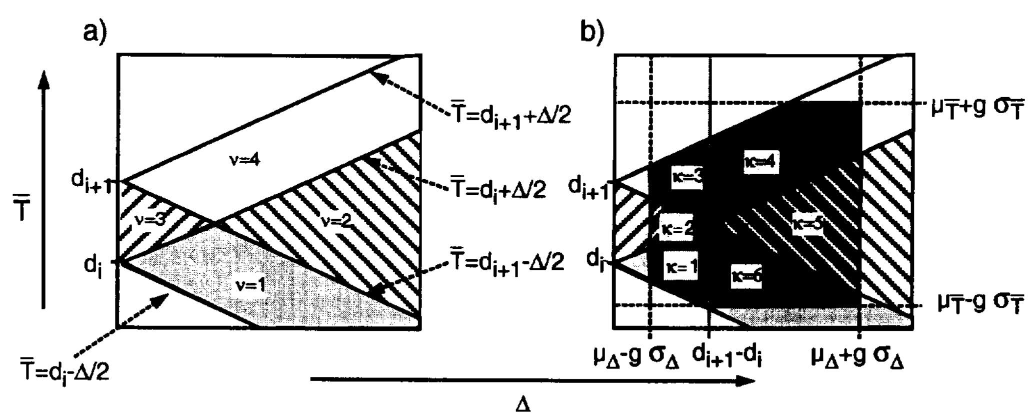

Fig. 3. Combinations of Δ and $\bar{T}$; (a) combinations of daily temperature amplitude Δ and daily temperature mean $\bar{T}$, which determine four different possibilities (areas Ω_v, $v = 1, \ldots, 4$) of the position of the daily temperature triangle relative to the temperature thresholds d_i and d_{i+1} (Fig. 2), and hereby of the formulation of the daily dependence function $D\tilde{E}P_i$ in Eq. (A.3.7); and (b) Final integration area in the $(\bar{T}, \Delta)$-plane: It consists of the six areas Ω_κ where the expected value of the daily dependence function is evaluated, i.e. the double integral (Eq. (A.3.11)) is solved. The four areas of (a) are further bounded by the values of temperature means $\mu_{\bar{T}} \pm 2\sigma_{\bar{T}}$ and amplitudes $\mu_\Delta \pm 2\sigma_\Delta$, outside of which the density function approximation is 0 (Fig. 1c). The areas Ω_1 and Ω_4 of (a) are split by the line $\Delta = d_{i+1} - d_i$.

In the following we transform the resulting function for $D\tilde{E}P_i$ (Eq. (A.3.4)) to a general polynomial form in $\bar{T}$ and Δ which is more convenient for the numerical evaluation and particularly for the subsequent evaluation of the expected value.

The solution for the general vth case of Eq. (A.3.4) can be expressed by

$$D\tilde{E}P_{i,v}(k_1, k_2, \bar{T}, \Delta) = \begin{cases} \beta_i((1-k_1) + k_1\gamma_{i+1} - k_2\gamma_i) + \\ \quad \alpha_i \dfrac{\Delta}{2}((1-k_1) + k_1\gamma_{i+1}^2 - k_2\gamma_i^2), & d_i - \dfrac{\Delta}{2} \le \bar{T} \le d_{i+1} + \dfrac{\Delta}{2} \\ 0, & \text{else} \end{cases} \quad \text{(A.3.6)}$$

with

$$\alpha_i = \frac{dep(d_{i+1} - dep(d_i)}{d_{i+1} - d_i},\ \beta_i = dep(d_i) + \alpha_i\left(\bar{T} - \frac{\Delta}{2} - d_i\right),\ \gamma_i = \frac{d_i - \bar{T} + \frac{\Delta}{2}}{\Delta}.$$

Table 3
Values of the integral boundaries $\lrcorner$ and $\urcorner$ depending on position of T_{min} and T_{max} with respect to d_i and d_{i+1}, obtained with $\lrcorner_{i,1} = \max(0, t_{i,1})$, $\urcorner_{i,1} = \min(t_{i+1,1}, t_{max})$, $\lrcorner_{i,2} = \max(t_{max}, t_{i+1,2})$, and $\urcorner_{i,2} = \min(t_{i,2}, 1)$ and Eq. (A.3.5).

v	$T_{min} >$	$T_{min} <$	$T_{max} >$	$T_{max} <$	$\lrcorner_{i,1}$	$\urcorner_{i,1}$	$\lrcorner_{i,2}$	$\urcorner_{i,2}$
1		d_i	d_i	d_{i+1}	$\gamma_i\ t_{max}$	t_{max}	t_{max}	$1-\gamma_i(1-t_{max})$
2		d_i	d_{i+1}		$\gamma_i\ t_{max}$	$\gamma_{i+1}\ t_{max}$	$1-\gamma_{i+1}\ (1-t_{max})$	$1-\gamma_i\ (1-t_{max})$
3	d_i	d_{i+1}	d_i	d_{i+1}	0	t_{max}	t_{max}	1
4	d_i	d_{i+1}	d_{i+1}		0	$\gamma_{i+1}\ t_{max}$	$1-\gamma_{i+1}\ (1-t_{max})$	1

The values γ_i are defined by $\gamma_i = (d_i - \bar{T} + \Delta/2)/\Delta$.

Table 4
Values of f_{α_i} and f_{β_i} of $D\tilde{E}P_i$ depending on the combination of T_{min} and T_{max} obtained by substituting the values for ⌋ and ⌉ of Table 3 in Eq. (A.3.4)

v	f_{β_i}	f_{α_i}	k_1	k_2
1	$1-\gamma_i$	$0.5\ (1-\gamma_i^2)$	0	1
2	$\gamma_{i+1}-\gamma_i$	$0.5\ (\gamma_{i+1}^2-\gamma_i^2)$	1	1
3	1	0.5	0	0
4	γ_{i+1}	$0.5\gamma_{i+1}^2$	1	0

The values γ_i are defined by $\gamma_i=(d_i-\bar{T}+\Delta/2)/\Delta$.

The parameters k_1 and k_2 (Table 4) depend thereby on the combination of $\bar{T}$ and Δ which differ for the four cases $v=1, \ldots, 4$ (Fig. 3a).

By backsubstitution of γ_i and β_i $D\tilde{E}P_{i,v}$ $(k_1, k_2, \bar{T}, \Delta)$ leads to a polynomial in $\bar{T}$ and a rational function in Δ, which can be expressed (e.g. with the help of symbolic calculation software) by

$$D\tilde{E}P_{i,v}(k_1,k_2,\bar{T},\Delta)=\sum_{j=1}^{3}\sum_{l=1}^{3}\xi_{j,l}(k_1,k_2)\bar{T}^{l-1}\Delta^{j-2} \tag{A.3.7}$$

with the elements $\xi_{j,l}$ $(k_1, k_2)\in C_D$ of the coefficient matrix

$$C_D=\begin{pmatrix} \rho_i(d_{i+1}k_1-d_ik_2)+\alpha_i\left(\frac{d_{i+1}^2k_1}{2}-\frac{d_i^2k_2}{2}\right) & \rho_i(-k_1+k_2) & \frac{\alpha_i(-k_1+k_2)}{2} \\ \rho_i\left(1-\frac{3k_1}{2}+\frac{k_2}{2}\right)+\alpha_i(d_ik_2-d_{i+1}k_1) & \frac{\alpha_i(2-k_1-k_2)}{2} & 0 \\ \frac{3\alpha_i(k_1-k_2)}{8} & 0 & 0 \end{pmatrix} \tag{A.3.8}$$

and $\rho_i=\mathrm{dep}(d_i)-\alpha_id_i$.

Example A.1. The case of Fig. 2b yields

$$T_{max}=\bar{T}+\frac{\Delta}{2}>d_{i+1}\Rightarrow\bar{T}>d_{i+1}-\frac{\Delta}{2},$$

$$T_{min}=\bar{T}-\frac{\Delta}{2}<d_i\Rightarrow\bar{T}<d_i+\frac{\Delta}{2},$$

and corresponds thus, in Fig. 3a to area Ω_2. Therefore, from Table 4 we get the values $f_{\beta_i}=\gamma_{i+1}-\gamma_i$, $f_{\alpha_i}=0.5(\gamma_{i+1}^2-\gamma_i^2)$, $k_1=1$, and $k_2=1$. With these values, the ith part of the approximated dependence function yields

$$D\tilde{E}P_{i,2}(1,1,\bar{T},\Delta)=\{\beta_i(\gamma_{i+1}-\gamma_i)+0.5\alpha_i\Delta(\gamma_{i+1}^2-\gamma_i^2)\}_{\text{in representation of equation (A.3.4)}}$$
$$=\left\{\frac{1}{\Delta}(\rho_i(d_{i+1}-d_i)+0.5\alpha_i(d_{i+1}^2-d_i^2))+\alpha_i(d_i-d_{i+1})\right\}_{\text{as polynomial (cf. equation (A.3.7).}}$$

Expected value: now, with the approximated daily dependence function integral $D\tilde{E}P_i$ we are able to determine the expected values by using the approaches DAT (Eq. (2.1.6)), EDDT1, and EDDT2 (Eq. (2.1.7)).

(1) For approach DAT, the arguments $\bar{T}$ and Δ in Eq. (A.3.7) are replaced by their mean values $\mu_{\bar{T}}$ and μ_Δ. Analogously to Eq. (2.1.6) we get

$$E[D\tilde{E}P_i] \simeq D\tilde{E}P_{i,v}(k_1, k_2, \mu_{\bar{T}}, \mu_\Delta). \tag{A.3.9}$$

The number v and the values k_1 and k_2 depend on the position of $\mu_{\bar{T}}$ and μ_Δ in the $(\bar{T}, \Delta)$ – plane relatively to the actual values of d_i and d_{i+1} (Fig. 3a and Table 4). To obtain the total expected values, the $(E[D\tilde{E}P_i])_{i>0}$ have to be summed over all i, i.e.

$$E[D\tilde{E}P] = \sum_i E[D\tilde{E}P_i] \simeq \sum_i D\tilde{E}P_{i,v(i)}(k_1, k_2, \mu_{\bar{T}}, \mu_\Delta) \tag{A.3.10}$$

(2) For approaches EDDT1 and EDDT2 we substitute in Eq. (2.1.7) the daily temperature dependence integral $DEP(\bar{T}, \Delta)$ and the probability densities $p_{\bar{T}}(y)$ and $p_\Delta(z)$ by their approximations $D\tilde{E}P_i$ (Eq. (A.3.7)), $\tilde{p}_{\bar{T}}(y)$ and $\tilde{p}_\Delta(z)$ (Eq. (2.2.3)) and obtain

$$E[D\tilde{E}P_i] \simeq \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D\tilde{E}P_i(k_1, k_2, y, z)\tilde{p}_{\bar{T}}(y)\,\mathrm{d}y\,\tilde{p}_\Delta(z)\,\mathrm{d}z. \tag{A.3.11}$$

Because $D\tilde{E}P_i$ is different in the four areas Ω_v, $v = 1, \ldots, 4$ (Eq. (A.3.6)) in the $(\bar{T}, \Delta)$-plane as shown in Fig. 3a, we now have to solve the resulting integral over each of these four domains. These integration domains are additionally bounded by the values $\mu_{\bar{T}} \pm 2\sigma_{\bar{T}}$ and $\mu_\Delta + 2\sigma_\Delta$ (depending on $\bar{T}$ and Δ) which define the interval $[\mu_x - 2\sigma_x, \mu_x + 2\sigma_x]$ where the approximation of the density functions is $\neq 0$ (Fig. 1c). Furthermore, the areas Ω_1 and Ω_4 are split by the line $\Delta = d_{i+1} - d_i$ to obtain as integration boundaries of the inner integral continuous functions of the outer integration variable z. The resulting six integration domains Ω_κ, $\kappa = 1, \ldots, 6$ are shown in Fig. 3b. The resulting boundaries together with the values k_1 and k_2 are listed in Table 5. For the approximation of the expected value we get now

$$E[D\tilde{E}P_i] \simeq \sum_{\kappa=1}^{6} \left\{ \iint_{\Omega_\kappa} \{D\tilde{E}P_i(k_1, k_2, y, z)\tilde{p}_{\bar{T}}(y)\tilde{p}_\Delta(z)\}_{P_\kappa}\,\mathrm{d}y\,\mathrm{d}z \right\}_{\tilde{E}_\kappa[D\tilde{E}P_i]} = \sum_{\kappa=1}^{6} \int_{\rfloor_{\Delta,\kappa}}^{\rceil_{\Delta,\kappa}} \int_{\rfloor_{\bar{T},\kappa}}^{\rceil_{\bar{T},\kappa}} P_\kappa\,\mathrm{d}y\,\mathrm{d}z.$$

Thereby, the integrand P_κ is a polynomial of 4th order in y, and of 3rd order in z because the density function approximations $\tilde{p}_{\bar{T}}(y)$ and $\tilde{p}_\Delta(z)$ (Eq. (2.2.3)) can be written as polynomials

$$\tilde{p}_{\bar{T}}(y) = \sum_{n=1}^{3} \zeta_n y^{n-1}, \qquad \tilde{p}_\Delta(z) = \sum_{m=1}^{3} \rho_m z^{m-1}$$

with the coefficients

Table 5
The integration boundaries $\rfloor_{\Delta,\kappa}$, $\rceil_{\Delta,\kappa}$, $\rfloor_{\bar{T},\kappa}$, and $\rceil_{\bar{T},\kappa}$ and the values k_1 and k_2 depend on the integration domains Ω_κ, which are defined by the combination of Δ and $\bar{T}$ (Fig. 3) and by the boundaries of the parabola approximating the density functions of $\bar{T}$ and Δ (Fig. 1c)

κ	$\rfloor_{\Delta,\kappa}$	$\rceil_{\Delta,\kappa}$	$\rfloor_{\bar{T},\kappa}$	$\rceil_{\bar{T},\kappa}$	k_1	k_2
1	$\max(0, \vdash_\Delta)$	$\min(d_{i+1} - d_i, \dashv_\Delta)$	$\max(d_i - \Delta/2, \vdash_{\bar{T}})$	$\min(d_i + \Delta/2, \vdash_{\bar{T}})$	0	1
2	$\max(0, \vdash_\Delta)$	$\min(d_{i+1} - d_i, \dashv_\Delta)$	$\max(d_i + \Delta/2, \vdash_{\bar{T}})$	$\min(d_{i+1} - \Delta/2, \vdash_{\bar{T}})$	0	0
3	$\max(0, \vdash_\Delta)$	$\min(d_{i+1} - d_i, \dashv_\Delta)$	$\max(d_{i+1} - \Delta/2, \vdash_{\bar{T}})$	$\min(d_{i+1} + \Delta/2, \dashv_{\bar{T}})$	1	0
4	$\max(d_{i+1} - d_i, \vdash_\Delta)$	$\dashv_\Delta$	$\max(d_i - \Delta/2, \vdash_{\bar{T}})$	$\min(d_i + \Delta/2, \dashv_{\bar{T}})$	0	1
5	$\max(d_{i+1} - d_i, \vdash_\Delta)$	$\dashv_\Delta$	$\max(d_i + \Delta/2, \vdash_{\bar{T}})$	$\min(d_{i+1} - \Delta/2, \dashv_{\bar{T}})$	1	1
6	$\max(d_{i+1} - d_i, \vdash_\Delta)$	$\dashv_\Delta$	$\max(d_{i+1} - \Delta/2, \vdash_{\bar{T}})$	$\min(d_{i+1} + \Delta/2, \dashv_{\bar{T}})$	1	0

We use the abbreviations: $\vdash_\Delta = \mu_\Delta - g \cdot \sigma_\Delta$, $\dashv_\Delta = \mu_\Delta + g \cdot \sigma_\Delta$, $\vdash_{\bar{T}} = \mu_{\bar{T}} - g \cdot \sigma_{\bar{T}}$; and $\dashv_{\bar{T}} = \mu_{\bar{T}} + g \cdot \sigma_{\bar{T}}$.

$$\zeta_n \in C_{\tilde{T}} = \begin{pmatrix} \dfrac{3}{4g\sigma_{\tilde{T}}} - \dfrac{3\mu_{\tilde{T}}^2}{4g^3\sigma_{\tilde{T}}^3} \\ \dfrac{3\mu_{\tilde{T}}}{2g^3\sigma_{\tilde{T}}^3} \\ \dfrac{-3}{4g^3\sigma_{\tilde{T}}^3} \end{pmatrix} \quad \text{and} \quad \rho_m \in C_\Delta = \begin{pmatrix} \dfrac{3}{4g\sigma\Delta} - \dfrac{3\mu_\Delta^2}{4g^3\sigma_\Delta^3} \\ \dfrac{3\mu_\Delta}{2g^3\sigma_\Delta^3} \\ \dfrac{-3}{4g^3\sigma_\Delta^3} \end{pmatrix}$$

and therefore we can write

$$P_\kappa = D\tilde{E}P_i(k_1, k_2, y, z)\tilde{p}_{\tilde{T}}(y)\tilde{p}_\Delta(z) = \left(\sum_{j=1}^{3}\sum_{l=1}^{3} \xi_{j,l}(k_1, k_2)y^{l-1}z^{j-2}\right)\left(\sum_{n=1}^{3} \zeta_n y^{n-1}\right)\left(\sum_{m=1}^{3} \rho_m z^{m-1}\right)$$

$$= \sum_{j,l,m,n=1}^{3} \underbrace{\{\xi_{j,l}(k_1, k_2)\rho_m\zeta_n\}}_{c_{j,l,m,n}(k_1,k_2)} y^{l+n-2}z^{j+m-3}$$

$$\Rightarrow E[D\tilde{E}P_i] \simeq \sum_{\kappa=1}^{6} \int_{\rfloor_{\Delta,\kappa}}^{\rceil_{\Delta,\kappa}} \int_{\rfloor_{\tilde{T},\kappa}}^{\rceil_{\tilde{T},\kappa}} \sum_{j,l,m,n=1}^{3} c_{j,l,m,n}(k_1, k_2)y^{l+n-2}z^{j+m-3}\,\mathrm{d}y\,\mathrm{d}z. \qquad \text{(A.3.12)}$$

This integral can be solved with some calculation effort, e.g. with the help of symbolic calculation software, because the integrand as well as the bounds of the inner integral are polynomials.

An example is given at the end of this section.

Summarized, the expected value of *dep*(T) is determined by summing the expected values of each of the linear pieces of the dependence function. The expected value of the ith linear piece is calculated by first determining the thresholds d_i and d_{i+1} of this piece. Then the integrals over each of the areas Ω_κ, have to be solved and summed. For each area, κ determines the values k_1 and k_2 and the integration borders $\rfloor_{\Delta,\kappa}$, $\rceil_{\Delta,\kappa}$, $\rfloor_{\tilde{T},\kappa}$, and $\rceil_{\tilde{T},\kappa}$, (Table 5). With the k-values, the coefficients $\xi_{j,l}(k_1, k_2)$ can now be determined from matrix C_D (Eq. (A.3.8)) and with this information, the double integral (Eq. (A.3.12)) can be solved.

The following example explains this procedure for $\kappa = 5$.

Example A.2. For the case $\kappa = 5$, which corresponds to Fig. 2b, we get with $d_{i+1} - d_i > \mu_\Delta - g\cdot\sigma_\Delta$, $d_i + \Delta/2 > \mu_T - g\cdot\sigma_T$, and $d_{i+1} - \Delta/2 < \mu_T + g\cdot\sigma_T$, from Table 5 the values $k_1 = 1$, $k_2 = 1$, $\rfloor_{\Delta,5} = d_{i+1} - d_i$, $\rceil_{\Delta,5} = \mu_\Delta + g\cdot\sigma_\Delta$, $\rfloor_{\tilde{T},5} = d_i + \Delta/2$ and $\rceil_{\tilde{T},5} = d_{i+1} - \Delta/2$. Hence the 5th summand of Eq. (A.3.12) is given by

$$\tilde{E}_5[\mathrm{D\tilde{E}P}_i] = \int_{d_{i+1}-d_i}^{\mu_\Delta+g\sigma_\Delta} \int_{d_i+z/2}^{d_{i+1}-z/2} \sum_{j,l,m,n=1}^{3} c_{j,l,m,n}(1, 1)y^{l+n-2}z^{j+m-3}\,\mathrm{d}y\,\mathrm{d}z$$

$$= \int_{d_{i+1}-d_i}^{\mu_\Delta+g\sigma_\Delta} \sum_{j=1}^{3}\sum_{m=1}^{3} \rho_m z^{j+m-3} \int_{d_i+z/2}^{d_{i+1}-z/2} \sum_{l=1}^{3}\sum_{n=1}^{3} \zeta_n\xi_{j,l}(1, 1)y^{l+n-2}\,\mathrm{d}y\,\mathrm{d}z$$

because only $\xi_{1,1}$, $\xi_{2,1} \neq 0$ we get thus,

$$= \int_{d_{i+1}-d_i}^{\mu_\Delta+g\sigma_\Delta} \sum_{j=1}^{2}\sum_{m=1}^{3} \rho_m z^{j+m-3}\xi_{j,1}(1, 1) \int_{d_i+z/2}^{d_{i+1}-z/2} \sum_{n=1}^{3} \zeta_n y^{1+n-2}\,\mathrm{d}y\,\mathrm{d}z$$

$$= \int_{d_{i+1}-d_i}^{\mu_\Delta+g\sigma_\Delta} \sum_{j=1}^{2}\sum_{m=1}^{3} \rho_m z^{j+m-3}\xi_{j,1}(1, 1) \sum_{n=1}^{3} \frac{\zeta_n}{n}\left(\left(d_{i+1} - \frac{z}{2}\right)^n - \left(d_i + \frac{z}{2}\right)^n\right)\mathrm{d}z$$

$$= \int_{d_{i+1}-d_i}^{\mu_\Delta+g\sigma_\Delta} \sum_{j=1}^{2}\sum_{m=1}^{3} \rho_m z^{j+m-3}\xi_{j,1}(1, 1)$$

$$\left(\left\{\left(\zeta_1(d_{i+1}-d_i)+\frac{\zeta_2}{2}(d_{i+1}^2-d_i^2)+\frac{\zeta_3}{3}(d_{i+1}^3-d_i^3)\right)\right\}_{a_0}\right.$$
$$\left.+\left\{\left(-\zeta_1+\frac{\zeta_2}{2}(-d_{i+1}-d_i)+\frac{\zeta_3}{2}(-d_{i+1}^2-d_i^2)\right)\right\}_{a_1}z+\left\{\frac{\zeta_3}{4}(d_{i+1}-d_i)\right\}_{a_2}z^2+\left\{\frac{\zeta_3}{12}\right\}_{a_3}z^3\right)\mathrm{d}z$$
$$=\int_{d_{i+1}-d_i}^{\mu_\Delta+g\sigma_\Delta}\sum_{j=1}^{2}\sum_{m=1}^{3}\rho_m\xi_{j,1}(1,1)\sum_{n=0}^{3}a_n z^{j+m+n-3}\,\mathrm{d}z$$
$$=\sum_{j=1}^{2}\sum_{m=1}^{3}\sum_{n=0}^{3}\frac{p_m\xi_{j,1}(1,1)a_n}{j+m+n-2}((\mu_\Delta+g\sigma_\Delta)^{j+m+n-2}-(d_{i+1}-d_i)^{j+m+n-2}).$$

References

Aceituno, P., 1979. Statistical formula to estimate heating or cooling degree-days. Agric. For. Meteorol., 20: 227–232.

Allen, J., 1976. A modified sine wave method for calculating degree days. Environ. Entomol., 5: 388–396.

Baskerville, G. and Emin, P., 1969. Rapid estimation of heat accumulation from maximum and minimum temperatures. Ecology, 50: 514–517.

Blago, N. and Dickler, E., 1990. Effectiveness of the Californian prognosis model 'BUGOFF2' for *Cydia pomonella L.* (Lepidoptera, Tortricidae) under Central European conditions. Acta Hortic., 276: 53–62.

Bugmann, H., 1994. On the ecology of mountainous forests in a changing climate: A simulation study. PhD thesis. Swiss Federal Institute of Technology Zurich. Diss. No. 10 638.

Fischlin, A., Bugmann, H. and Gyalistras, D., 1994. Sensitivity of a forest ecosystem model to climate parametrization schemes. Environ. Pollut., 87: 267–282.

Kirsta, Y.B. and Tarabrin, V., 1994. Real biological time and its calculation in wheat. Ecol. Model., 71 259–267.

Lindsey, A. and Newman, J., 1956. Use of official weather data in spring time—Temperature analysis of an Indiana phenological record. Ecology, 37: 812–823.

Lischke, H., 1991. Ein Modell zur Simulation der Populationsdynamik des Apfelwicklers (*Cydia Pomonella L.* (Lepidoptera, Tortricidae)). PhD thesis. University of Heidelberg.

Lischke, H., 1992. A model to simulate the population dynamics of the codling moth (*Cydia pomonella L.* (Lepidoptera, Torticidae)): Parameter estimation and sensitivity analysis. Acta Hort., 313: 331–338.

Lischke, H. and Blago, N., 1990. A model to simulate the population dynamics of the codling moth (*Cydia pomonella L.* (Lepidoptera, Torticidae)): Development and male moth flight. Acta Hort., 276: 43–52.

Lischke, H., Loeffler, T.J. and Fischlin, A., 1996a. Aggregation of individual tress and patches in forest succession models: Capturing variability with height structured random dispersions. Technical Report 28. Systems Ecology, Institute of Terrestrial Ecology, ETH Zurich. Grabenstr.3, CH-8952 Schlieren, Switzerland.

Lischke, H., Loeffler, T. and Fischlin, A., 1996b. Calculating temperature dependence over long time periods: A comparison of methods. Technical Report 27. Systems Ecology, Institute of Terrestrial Ecology, ETH Zurich. Grabenstr.3, CH-8952 Schlieren, Switzerland.

Mann, M., Park, J. and Bradley, R., 1995. Global interdecadal and century-scale climate oscillations during the past five centuries. 378: 266–270.

Parton, W. and Logan, J.A., 1981. A model for diurnal variation in soil and air temperature. Agric. Meteorol., 23: 205–216.

Perruchoud, D. and Fischlin, A., 1995. The response of the carbon cycle in undisturbed forest ecosystems to climate change: A review of plant-soil models. J. Beogeogr., 22: 2603–2618.

Prentice, I., Sykes, M. and Cramer, W., 1993. A simulation model for the transient effects of climate change on forest landscapes. Ecol. Model. 65: 51–70.

Sharpe, P. and DeMichele, D., 1977. Reaction kinetics of poikilotherm development. J. Theor. Biol., 64: 649–670.

Stinner, R., Gutierrez, A. and Butler, G., 1974. An algorithm for temperature-dependent growth rate simulation. Can. Entomol., 106: 519–524.

Wagner, T., Wu, H., Sharpe, P., Schoolfield, R. and Coulson, R., 1984. Modeling insect development rates: A literature review and application of a biophysical model. Ann. Entomol. Soc. Am., 77: 208–225.

Worner, S., 1988. Evaluation of diurnal temperature models and thermal summation in New Zealand. J. Econ. Entomol., 81: 9–13.