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Derivation of methods

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Calculating temperature dependence over long time periods: Derivation of methods

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Abstract

Rates of ecological processes are usually influenced by temperature. For simplicity and efficiency of ecosystem models it is often necessary to summarise information about temperature dependence from short, e.g. hourly, time intervals over longer, e.g. monthly, time periods, i.e. to calculate long term expected values of dependence functions. This aim can seldom be achieved by applying the temperature function to the mean temperature, because temperature dependencies are in many cases nonlinear. Therefore, we derived newly seven methods for such a temporal aggregation of temperature dependence. The methods determine the expected value interpreting either hourly temperature, daily temperature mean, or daily temperature mean and amplitude as random variables. The dependence function hereby is approximated by a piecewise linear function, the daily temperature course by a triangle and the density function of the normal distribution by a parabola.
The resulting methods cover a range of temperature input data resolutions: monthly mean or standard deviation or both of either hourly temperatures, daily temperature extrema, daily temperature means and amplitudes, or only daily temperature means. The methods can be applied to all types of dependence functions, in particular to nonlinear ones.

Key words: temperature dependence, physiological time, modeling, aggregation, approximation, temperature time series

Contents

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1 Introduction

Many biologically or ecologically relevant processes are temperature dependent. This holds for development processes of poikilothermic organisms, as e.g. the maturing of insects or the growth of plants. Also processes used to synchronise an organism's life-cycle to seasonally changing environmental conditions, such as insect diapause, seed vernalisation or timing of tree bud rest break are at least partly regulated by temperature. The functions $dp(T)$ by which these processes depend on temperature T are usually nonlinear. Cummulative effects of temperature on such biological processes can be measured by means of the integral

$$M(t,t_0) \underset{\text{Def.}}{:=} \int_{t_0}^{t} dp(T(\tau))d\tau \tag{1.0.1}$$

over a time interval (t_0, t). This integral is often referred to as "physiological time", "day-degree-sum", or "heat-unit-sum".

Temperature dependence plays also a major role in many ecological simulation models, ranging from pest prognosis models (e.g. BUGOFF2 (BLAGO AND DICKLER 1990) and APFWICK (LISCHKE AND BLAGO 1990, LISCHKE 1992), over crop phenology models (e.g. BIOTIME, (KIRSTA AND TARABRIN 1994)), to models examining the sensitivity of ecosystems to a potential climatic change, as e.g. the forest succession models FORSKA (PRENTICE ET AL. 1993), FORCLIM (BUGMANN 1994, FISCHLIN ET AL. 1994), and DISCFORM (LISCHKE ET AL. 1995A), where physiological time determines e.g. the growth and thus the competition of individual trees.

An imprecise formulation of the temperature dependence function can seriously influence the outcome of such models, depending on the model sensitivity to the regarded temperature dependence function. For example, an about 10% error of the temperature dependence of codling moth developement leads to an error of about 7 days in the simulations with a pest prognosis model (LISCHKE 1992) in Central Europe. Depending on the application, such an error might be untolerable.

The most exact approach is to calculate $M(t,t_0)$ by summing the actual values of the dependence function using temperature data in high temporal resolution, which reflect the diel and even higher frequency temperature fluctuations.

However, due to practical constraints as the lack of appropriate input data, long computation time, or the desire to keep a model as simple as possible, in many models a larger time step is chosen and temperature dependence is calculated by applying the temperature dependence function either to the mean temperatures, (e.g. monthly temperature means in FORCLIM) or to an interpolated temperature course (e.g. in FORSKA or theBIOTIME-model).

Yet, monthly or yearly temperature means or interpolations between means do not contain all information about the temperature variability in the regarded period, particularly not about the diel variation. If the dependence function is nonlinear, which is the case for many processes, such a simple approach can lead to a loss of precision in the model outcome.

To overcome this conflict between required precision and manageability, methods are necessary to calculate physiological time as precisely as needed using as much information of the available input data as possible . The methods should work on larger time scales, at least one day, preferably one month, year, or decade. i.e. aggregate the temperature dependence function from the small time scale of the input data to a larger time scale. This means, the methods summarise the information about temperature dependence from short time intervals over longer periods.

Several approaches exist to deal with this problem. **(1)** A possibility is to approximate the nonlinear dependence function using a linear one with a lower and an upper threshold, and to sum the daily values of this approximation as in BUGOFF2 or by calculating its expected value (ACEITUNO 1979). However, the use of such a linearly and monotonically increasing approximation instead of the original dependence function or a better matching nonlinear approximation as e.g. the sigmoid function proposed in (STINNER ET AL. 1974) or the biophysical models presented by Sharpe and DeMichele (1977) and Wagner et al. (1984) can lead to considerable loss of precision (BLAGO AND

DICKLER 1990).
(2) Other approaches apply the dependence function to an estimated daily temperature course, which has been approximated by models such as the triangulation method of Lindsey and Newman (1956), the single sine method by Baskervile and Emin (1969), the sine-sine-method of (ALLEN 1976), or the sine-exponential method of Parton and Logan (1981). However, tests of some of these methods by Worner (1988) did not show a satisfactory precision for all tested sites. Moreover, if it is nonlinear, the temperature dependence function can not be evaluated in one step for each day but has to be applied to hourly values of the approximated temperature course, so that no computing time is saved compared to the use of the original hourly input data. The computational costs for solving the integral $M(t, t_0)$ over one day can only be reduced for uncomplicated, e.g. linear dependence functions as in BUGOFF2.
(3) Empirical correction functions of the temperature dependence as used by Allen (1976) or Bugmann (1994) on the other hand confine the model application to the regions where those functions have been estimated.

Summarised, the above listed approaches are either restricted to a special, often linear type of dependence function, to a certain length of the aggregation period, or to a certain kind of input data, even if more detailed information about the temperature course in the aggregation interval is available. Or they have to be combined with empirical correction terms to yield satisfying results.

The aim of this paper is to derive several approaches for temperature dependence aggregation

- which are applicable for general, i.e. nonlinear temperature dependence functions;
- for temperature input data of different resolutions;
- which are able to use as much information as possible in the available temperature input data,
- and to work with arbitrarily large time steps, ranging from days to decades;
- and which are generally formulated and therefore extendible to other fields of dependence functions;

2 Derivation of Methods

2.1 Principles

In this section the approximation principles of the methods are described. The same general idea is underlying all described approaches. If with a certain temperature x, $p_{T,act}(x)$ is the relative frequency in the aggregation interval (t_0, t), e.g. one month, then the physiological time $M(t, t_0)$ (cf. (1.0.1)) can be expressed by the integral over the dependence function of x multiplied with its absolute frequency by

$$\begin{aligned} M(t, t_0) & \underset{\text{Def.}}{:=} \int_{t_0}^{t} dep(T(\tau))d\tau \\ & = (t - t_0) \int_{-\infty}^{\infty} p_{T,act}(x) dep(x) dx \\ & = (t - t_0) E[dep(T)]. \end{aligned}$$

Thereby $E[dep(T)]$ is the expected value of the temperature dependence function $dep(T)$. The problem is to find a reliable estimator for $E[dep(T)]$ in (t_0, t), given the mean value and standard deviation or only the mean value of temperature or related variables as e.g. temperature extrema.
In the following eight methods for the estimation of $E[dep(T)]$ are derived. The approximations used for the estimation and the exact algorithms are given in sections 2.2 and 2.3 respectively. The symbols are explained in tab. 5 in the appendix. For sake of simplicity we consider the aggregation from an

hourly to a monthly time interval, but the methods can also be applied for other aggregations from all time intervals of less than one day to larger ones, e.g. one year or decade.

2.1.1 Methods using the hourly temperature as random variable

We describe two approaches, abbreviated as DA and EDH respectively, which regard the hourly temperature as a normally distributed random variable with the density function $p_T(x)$.
In the widely used approach DA, the expected value is approximated by applying the dependence function directly to the mean temperature value μ_T in the regarded period, i.e.

$$E[dep(T)] \simeq dep(\mu_T). \tag{2.1.1}$$

In approach EDH the expected value $E[dep(T)]$ of the hourly values of the temperature dependence is calculated explicitly by

$$T \sim N(\mu_T, \sigma_T) \Rightarrow \quad p_T(x) = \frac{e^{-(x-\mu_T)^2/2\sigma_T^2}}{\sigma_T\sqrt{2\pi}} \tag{2.1.2}$$

$$E[dep(T)] = \int_{-\infty}^{\infty} dep(x)p_T(x)dx. \tag{2.1.3}$$

2.1.2 Methods using daily temperature mean, amplitude, and extrema as random variables

If no hourly input data, but data about the daily temperature means, amplitudes, or extrema are available, the following six methods EDHT1, ETHT2, EDM and DAT, EDDT1, EDDT2 can be used.

Methods approximating the statistical parameters of hourly temperatures **(1)** Approach EDHT1 applies the same algorithm as EDH, but estimates the mean μ_T and standard deviation σ_T (cf. (2.3.2)) of the hourly temperatures from the means $\mu_{\bar{T}}, \mu_\Delta$ and standard deviation $\sigma_{\bar{T}}, \sigma_\Delta$ of the daily mean $\bar{T}$ and amplitude Δ.
(2) Approach EDHT2 corresponds to EDHT1, with the difference of calculating the daily temperature mean $\bar{T}$ and amplitude Δ from the daily extrema by $\bar{T} \simeq \bar{T}_m = \frac{T_{min}+T_{max}}{2}$ and $\Delta = T_{max} - T_{min}$.
(3) In approach EDM the expected value is calculated explicitly as in EDH, but with the daily mean temperature $\bar{T}$ as random variable, which corresponds to the assumption that hourly and daily mean temperatures have a similar variance.

$$\bar{T} \sim N(\mu_{\bar{T}}, \sigma_{\bar{T}}) \Rightarrow \quad p_{\bar{T}}(y) = \frac{e^{-(y-\mu_{\bar{T}})^2/2\sigma_{\bar{T}}^2}}{\sigma_{\bar{T}}\sqrt{2\pi}}$$

$$E[dep(T)] = \int_{-\infty}^{\infty} dep(y)p_{\bar{T}}(y)dy. \tag{2.1.4}$$

Methods approximating the daily temperature dependence function In a first step the temperature course $T(t)$ for each day is approximated by a function $\widetilde{T}(t, \bar{T}, \Delta)$ (cf. (2.2.2)) of the daily average temperature $\bar{T}$ and daily temperature amplitude Δ. Then the daily integral $DEP(\bar{T}, \Delta)$ of the temperature dependence function $dep(T)$ applied to this approximated temperature course $\widetilde{T}(t, \bar{T}, \Delta)$ is evaluated (cf. (2.3.5) and (2.3.6)) by

$$DEP(\bar{T}, \Delta) \underset{\text{Def.}}{:=} \int_0^1 dep\left(\widetilde{T}(\tau, \bar{T}, \Delta)\right) d\tau \simeq \int_0^1 dep\,(T(\tau))\, d\tau \tag{2.1.5}$$

for each day normalised to the interval $(0, 1)$ of the period (t_0, t). In a second step the expected value $E[DEP(\bar{T}, \Delta)]$ of $DEP(\bar{T}, \Delta)$ for all days in (t_0, t) is determined.
(1) Approach DAT approximates $E[DEP(\bar{T}, \Delta)]$ by applying DEP to the average daily temperature

course, which is characterised by the average daily temperature mean $\mu_{\bar{T}}$ and the average daily temperature amplitude μ_Δ, i.e.

$$E[DEP(\bar{T},\Delta)] \simeq DEP(\mu_{\bar{T}},\mu_\Delta). \tag{2.1.6}$$

(2) In approach EDDT1 the expected value of $DEP(\bar{T},\Delta)$ is calculated, regarding daily temperature mean $\bar{T}$ and amplitude Δ as independently normally distributed random variables with means $\mu_{\bar{T}}$ and μ_Δ, standard deviations $\sigma_{\bar{T}}$ and σ_Δ, and density functions $p_{\bar{T}}(y)$ and $p_\Delta(z)$, which are defined analogously to eq. (2.1.2). The expected value $E[DEP(\bar{T},\Delta)]$ is defined by

$$E[DEP(\bar{T},\Delta)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} DEP(y,z)\ p_{\bar{T}}(y)\ dy\ p_\Delta(z)\ dz. \tag{2.1.7}$$

(3) Approach EDDT2 corresponds to EDDT1, except that the daily temperature mean $\bar{T}$ is calculated by $\bar{T} \simeq \bar{T}_m = \frac{T_{min}+T_{max}}{2}$.

2.2 Approximations

In this chapter the approximations underlying all derived methods are described. The integrals in

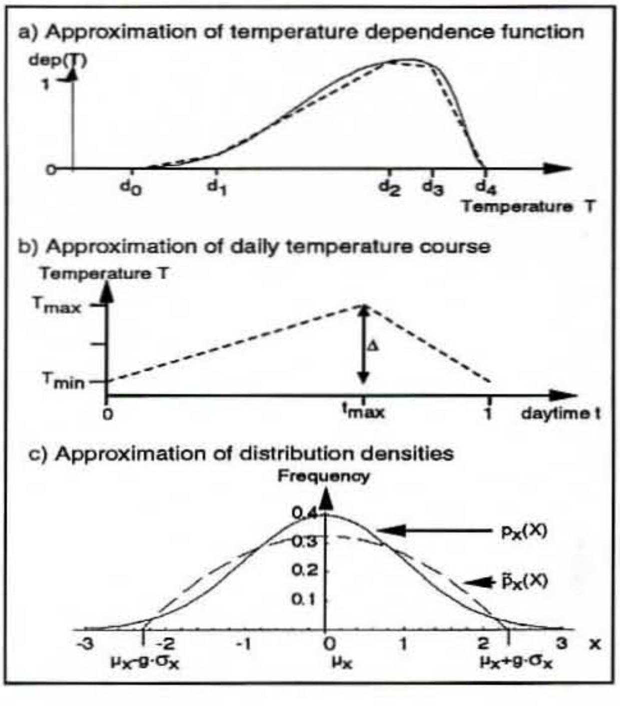

Figure 1: Approximations used in by the aggregation methods: **a)** Approximation of the temperature dependence function by a piecewise linear function, defined by a set of grid points d_i and the corresponding values of the dependence function. **b)** Approximation of the daily temperature course using a triangle between T_{min} and T_{max}. The maximum is reached at time t_{max}. **c)** Approximation of the density function $p_X(x)$ of the normal distribution by the parabola $\bar{p}_X(x)$, $X = \bar{T},\Delta$ (μ_X, σ_X are the mean and standard deviation).

eqs. (2.1.3), (2.1.4), and (2.1.7) are of the type $\int e^{-(x-a)^2} f(x)$ and thus not analytically soluble for all types of functions f. Therefore, we use the following approximations (cf. figs. 1a, 1b, and 1c):

- The temperature dependence function $dep(T)$ is approximated by a piecewise linear function $\widetilde{dep}(T)$ with the n_d grid points d_i.

$$\widetilde{dep}(T) = \begin{cases} dep(d_i) + (T-d_i)\frac{dep(d_{i+1})-dep(d_i)}{d_{i+1}-d_i}, & d_i \le T < d_{i+1} \\ 0 & \text{, else} \end{cases} \quad ,i = 0,\ldots,n_d-1. \tag{2.2.1}$$

In fig. 1a e.g. an approximation with four linear parts is shown.

- The daily temperature course is approximated by an asymmetric triangle $\tilde{T}(t)$ with the same minimum temperature at the beginning and end of the day (cf. fig. 1b) and a variable time point t_{max} of the maximum temperature.

$$
\begin{aligned}
T_{min} &= \bar{T} - \frac{\Delta}{2}; \; T_{max} = \bar{T} + \frac{\Delta}{2}; \\
\tilde{T}(t) &= \begin{cases} \bar{T} - \frac{\Delta}{2} + t\frac{\Delta}{t_{max}} & , 0 \le t < t_{max} \\ \bar{T} - \frac{\Delta}{2} + (1-t)\frac{\Delta}{1-t_{max}}, & t_{max} \le t < 1 \end{cases}
\end{aligned}
\tag{2.2.2}
$$

- In eq. (2.1.7) the normal distribution density functions $p_{\bar{T}}(y)$ and $p_{\Delta}(z)$ are each approximated by a polynomial $\tilde{p}$ of second order which are set to 0 for values $x, y < 0$ (in fig. 1c: $x < \mu_X - g\sigma_X$ and $x > \mu_X + g\sigma_X$, $X = \bar{T}, \Delta$). The parameter g determines the width and height of the parabola. In fig. 1c the approximation for $g = 2.3$ is shown.

$$
\tilde{p}_X(y) = \begin{cases} \frac{3}{g^3\sigma_X}\left(\left(\frac{g}{2}\right)^2 - \left(\frac{y-\mu_X}{2\sigma_X}\right)^2\right), & \mu_X - g\sigma_X < y < \mu_X + g\sigma_X \\ 0 & , \text{else} \end{cases}
\tag{2.2.3}
$$

with $X = \bar{T}, \Delta$. With these approximations we get (piecewise) polynomials for all functions in the integrals (2.1.3), (2.1.4), and (2.1.7). The so replaced integrals can be solved analytically with the help of symbolic calculation software as e.g. MATHEMATICA or MAPLE. In the following we refer to the aggregation methods using these approximations with a tilde.

2.3 Algorithms

In this section we derive in detail the algorithms, which are used to evaluate the new approaches (EDH, EDM, EDHT1, EDHT2, DAT, EDDT1, and EDDT2) the principles of which were presented in section 2.1.

The seven methods approximate the temperature dependence function $dep(T)$ by $\widetilde{dep}(T)$ (cf. (2.2.1)). Thus, this piecewise linear function $\widetilde{dep}(T)$ has to be defined suitably by the grid points $d_i, i = 0, \ldots, n_d - 1$. Then the expected value of $\widetilde{dep}(T)$ can be treated as a sum of the expected values of the different linear pieces, i.e. $E[\widetilde{dep}(T)] = \sum_{i=1}^{n_d} E[\widetilde{dep}_i(T)]$. In the following it is therefore sufficient to explain the evaluation of $E[\widetilde{dep}_i(T)]$.

2.3.1 Approach EDH

For approach EDH we substitute in eq. (2.1.3) the temperature dependence function $dep(T)$ with the approximation $\widetilde{dep}_i(T)$ (cf. (2.2.1)). In this way we get the approximated expected value $E[\widetilde{dep}_i(T)]$ of each i^{th} part of the dependence function as

$$
\begin{aligned}
E[\widetilde{dep}_i(T)] &= \int_{d_i}^{d_{i+1}} \widetilde{dep}_i(T)\frac{e^{-(x-\mu_T)^2/2\sigma_T^2}}{\sigma_T\sqrt{2\pi}}dx \\
&= \int_{d_i}^{d_{i+1}} \left(dep(d_i) + (x - d_i)\frac{dep(d_{i+1}) - dep(d_i)}{d_{i+1} - d_i} \right) \frac{e^{-(x-\mu_T)^2/2\sigma_T^2}}{\sigma_T\sqrt{2\pi}}dx \\
&= \int_{d_i}^{d_{i+1}} (\beta_i + \alpha_i x)e^{-(x-\mu_T)^2/\gamma}dx
\end{aligned}
$$

with $\alpha_i = \frac{dep(d_{i+1}) - dep(d_i)}{d_{i+1}-d_i}\frac{1}{\sigma_T\sqrt{2\pi}}$, $\beta_i = \frac{dep(d_i)}{\sigma_T\sqrt{2\pi}} - \alpha_i d_i$, $\gamma = 2\sigma_T^2$ which can be solved e.g. with the help of symbolic calculation software, because the function $\beta_i + \alpha_i x$ is linear in x. The solution yields

$$
E[\widetilde{dep}_i(T)] = 0.5\,(\alpha_i\mu_T + \beta_i)\sqrt{\frac{\gamma\pi}{2}}\sigma_T \quad \left(Erf\left(\frac{d_{i+1} - \mu_T}{\sqrt{\gamma}}\right) - Erf\left(\frac{d_i - \mu_T}{\sqrt{\gamma}}\right) \right) -
$$

$$0.5\,\alpha_i\gamma \qquad \left(e^{-(\mu_T-d_{i+1})^2/\gamma}-e^{-(\mu_T-d_i)^2/\gamma}\right) \tag{2.3.1}$$

with the errorfunction $Erf(x)$, which can be expressed by the series $\sum_{\kappa=1}^{\kappa_{max}} \frac{x^{2\kappa-1}(-1)^{\kappa-1}}{(\kappa-1)!(2\kappa-1)}$, stopping after κ_{max} iterations.

2.3.2 Approaches EDM, EDHT1, and EDHT2

The approaches EDM, EDHT1, and EDHT2 are all based on approach EDH (2.3.1) but differ in the way they estimate the data T, μ_T, and σ_T.
(1) For approach EDM in eq. (2.3.1) T, μ_T, and σ_T are replaced by $\bar{T}, \mu_{\bar{T}}$, and $\sigma_{\bar{T}}$.
(2) In approach EDHT2 in a first step the daily mean temperatures are approximated by $\bar{T} \simeq \bar{T}_m = \frac{T_{min}+T_{max}}{2}$ for each day. Hence we can approximate $\mu_{\bar{T}} \simeq \mu_{\bar{T}_m}$ and $\sigma_{\bar{T}} \simeq \sigma_{\bar{T}_m}$.
(3) Then in both approaches EDHT1 and EDHT2 we derive the mean μ_T and variance σ_T of the hourly temperatures from the mean $\mu_{\bar{T}}$ and μ_Δ and the variance $\sigma_{\bar{T}}$ and σ_Δ of the daily temperature means and amplitudes.
The approximation of μ_T is easy, since $\mu_T = \mu_{\bar{T}}$.
To obtain the approximation $\tilde{\sigma}_T$ of σ_T we assume that the temperature course $\tilde{T}_{day}(t)$ at a specific day follows a triangle, analogously to the approximation in fig. 1b) and eq. (2.2.2). The triangle is symmetric with maximum at noon and equal minima at the beginning and end of the day. Hence we get

$$\tilde{T}_{day}(t) = \begin{cases} \bar{T}_{day} - \frac{\Delta_{day}}{2} + t\frac{\Delta_{day}}{0.5} & , 0 \le t < 0.5 \\ \bar{T}_{day} - \frac{\Delta_{day}}{2} + (1-t)\frac{\Delta_{day}}{0.5}, & 0.5 \le t < 1 \end{cases}.$$

The variance σ_T of this approximated temperature course $\tilde{T}(t)$ during m days is given through the mean quadratic distance of each temperature value to the mean temperature by

$$\begin{aligned}
\tilde{\sigma}_T^2 &= \frac{1}{m}\sum_{day=1}^{m}\int_0^1 \left(\tilde{T}_{day}(t)-\mu_T\right)^2 dt \\
&= \frac{2}{m}\sum_{day=1}^{m}\int_0^{0.5} \left(\underbrace{\bar{T}_{day}-\frac{\Delta_{day}}{2}-\mu_T}_{\alpha} + t\underbrace{2\Delta_{day}}_{\beta}\right)^2 dt \\
&= \frac{2}{m}\sum_{day=1}^{m}\int_0^{0.5} \alpha^2+2\alpha\beta t+\beta^2t^2\, dt \\
&= \frac{1}{m}\sum_{day=1}^{m}\left(\alpha^2+\frac{\alpha\beta}{2}+\frac{\beta^2}{6}\right) \\
&= \frac{1}{m}\sum_{day=1}^{m}(\bar{T}_{day}-\mu_T)^2-\Delta_{day}(\bar{T}_{day}-\mu_T)+\frac{\Delta_{day}^2}{4}+\left(\bar{T}_{day}-\mu_T-\frac{\Delta_{day}}{2}\right)\Delta_{day}+\frac{4\Delta_{day}^2}{12} \\
&= \underbrace{\frac{1}{m}\sum_{day=1}^{m}(\bar{T}_{day}-\mu_T)^2}_{\sigma_{\bar{T}}^2} - \frac{1}{m}\sum_{day=1}^{m}\left(\underbrace{\Delta_{day}(-\bar{T}_{day}+\bar{T}_{day}-\mu_T+\mu_T)}_{0} + \Delta_{day}^2\left(\frac{1}{4}-\frac{1}{2}+\frac{1}{3}\right)\right) \\
&= \sigma_{\bar{T}}^2+\frac{1}{12}\frac{1}{m}\sum_{day=1}^{m}\Delta_{day}^2 \\
&= \sigma_{\bar{T}}^2+\frac{1}{12}\frac{1}{m}\sum_{day=1}^{m}\left(\Delta_{day}^2-2\Delta_{day}\mu_\Delta+\mu_\Delta^2+2\Delta_{day}\mu_\Delta-\mu_\Delta^2\right) \\
&= \sigma_{\bar{T}}^2+\frac{1}{12}\underbrace{\frac{1}{m}\sum_{day=1}^{m}(\Delta_{day}-\mu_\Delta)^2}_{\sigma_\Delta^2}+\frac{1}{12}\frac{1}{m}\sum_{day=1}^{m}\left(2\Delta_{day}\mu_\Delta-\mu_\Delta^2\right)
\end{aligned}$$

$$
\begin{aligned}
&= \sigma_{\bar{T}}^2 + \frac{1}{12}\sigma_\Delta^2 + \frac{2\mu_\Delta}{12m}\sum_{day=1}^{m}(\Delta_{day}) - \frac{1}{12}\mu_\Delta^2 \\
&= \sigma_{\bar{T}}^2 + \frac{1}{12}\sigma_\Delta^2 + \frac{2\mu_\Delta}{12}\mu_\Delta - \frac{1}{12}\mu_\Delta^2 \\
&= \sigma_{\bar{T}}^2 + \frac{1}{12}\sigma_\Delta^2 + \frac{1}{12}\mu_\Delta^2 .
\end{aligned}
$$

This gives the result

$$
\Rightarrow \widetilde{\sigma}_T = \sqrt{\frac{1}{12}(\sigma_\Delta^2 + \mu_\Delta^2) + \sigma_{\bar{T}}^2}. \tag{2.3.2}
$$

Then the algorithm EDH (2.3.1) is used with the obtained μ_T and $\widetilde{\sigma}_T$.

2.3.3 Approaches DAT, EDDT1, and EDDT2

For the following three methods first the daily value of the i^{th} part of the dependence function is calculated. Then the expected value of this daily value is approximated.

Daily dependence function integral The approximated daily integral $DEP(\bar{T},\Delta)$ (2.1.5) over the approximated dependence function part $\widetilde{dep}_i(T)$ (2.2.1) applied to the approximated temperature course $\bar{T}(t)$ (2.2.2) (illustrated by fig. 2) is given by

$$
\begin{aligned}
\widetilde{DEP}_i &= \int_0^1 \widetilde{dep}_i\left(\widetilde{T}(\tau)\right) d\tau \\
&= \int_0^1 dep(d_i) + \left(\widetilde{T}(\tau) - d_i\right) \underbrace{\frac{dep(d_{i+1}) - dep(d_i)}{d_{i+1} - d_i}}_{\alpha_i} d\tau \\
&= \int_{\rfloor_{i,1}}^{\rceil_{i,1}} \underbrace{dep(d_i) + \alpha_i\left(\bar{T} - \frac{\Delta}{2} - d_i\right)}_{\beta_i} + \alpha_i \tau \frac{\Delta}{t_{max}} d\tau + \\
&\quad \int_{\rfloor_{i,2}}^{\rceil_{i,2}} \underbrace{dep(d_i) + \alpha_i\left(\bar{T} - \frac{\Delta}{2} - d_i\right)}_{\beta_i} + \alpha_i(1-\tau)\frac{\Delta}{1 - t_{max}} d\tau \\
&= \beta_i\left(\rceil_{i,1} - \rfloor_{i,1}\right) + \frac{\alpha_i \Delta}{2t_{max}}\left(\rceil_{i,1}^2 - \rfloor_{i,1}^2\right) + \\
&\quad \beta_i\left(\rceil_{i,2} - \rfloor_{i,2}\right) + \frac{\alpha_i \Delta}{1 - t_{max}}\left(\rceil_{i,2} - \rfloor_{i,2} - \frac{1}{2}\left(\rceil_{i,2}^2 - \rfloor_{i,2}^2\right)\right) + \\
&= \beta_i \underbrace{\left(\rceil_{i,1} - \rfloor_{i,1} + \rceil_{i,2} - \rfloor_{i,2}\right)}_{f_{\beta_i}} + \\
&\quad \alpha_i \Delta \underbrace{\frac{(1 - t_{max})\left(\rceil_{i,1}^2 - \rfloor_{i,1}^2\right) - t_{max}\left(\rceil_{i,2}^2 - \rfloor_{i,2}^2\right) + 2\,t_{max}\left(\rceil_{i,2} - \rfloor_{i,2}\right)}{2\,t_{max}(1 - t_{max})}}_{f_{\alpha_i}} \\
&= \beta_i f_{\beta_i} + \alpha_i \Delta f_{\alpha_i}
\end{aligned} \tag{2.3.3}
$$

with $\rfloor_{i,1} = \max(0, t_{i,1})$, $\rceil_{i,1} = \min(t_{i+1,1}, t_{max})$, $\rfloor_{i,2} = \max(t_{max}, t_{i+1,2})$, and $\rceil_{i,2} = \min(t_{i,2}, 1)$.
Fig. 2 shows that $t_{i,1}$, $t_{i+1,1}$, $t_{i,2}$, and $t_{i+1,2}$ are the times when the temperature reaches d_i and d_{i+1}, the lower and upper threshold of the dependence function during the increasing respectively decreasing part of the approximated daily time course $\bar{T}(t)$. These times can be calculated by the

inverse function of eq. (2.2.2), i.e.

$$t = \begin{cases} t_{max} \frac{\tilde{T} - \bar{T} + \frac{\Delta}{2}}{\Delta} & , 0 \le t < t_{max} \\ 1 - (1 - t_{max}) \frac{\tilde{T} - \bar{T} + \frac{\Delta}{2}}{\Delta}, & t_{max} \le t < 1 \end{cases} . \tag{2.3.4}$$

The integration borders $\rceil$ and $\rfloor$ depend on T_{min} and T_{max} as well as on the thresholds d_i and d_{i+1}. According to whether the temperatures of the regarded day remain between these thresholds, cut them or lie outside of them, we get four different cases of (T_{min}, d_i) and (T_{max}, d_{i+1}) combinations, where $\widetilde{dep} \neq 0$ (cf. fig. 2), mentioned in (ALLEN 1976). Because $T_{min} = \bar{T} - \frac{\Delta}{2}$ and $T_{max} = \bar{T} + \frac{\Delta}{2}$, these four cases correspond to four combinations of $\bar{T}$ and Δ drawn as shaded areas $\Omega_\nu, \nu = 1, \ldots, 4$ in fig. 3a). The values of the integral boundaries $\rfloor_{i,1}, \rceil_{i,1}, \rfloor_{i,2}$, and $\rceil_{i,2}$ are given in tab. 1. The resulting values for f_{α_i} and f_{β_i} in eq. (2.3.3) for the four cases are listed in tab. 2.

ν	T_{min} $>$	T_{min} $<$	T_{max} $>$	T_{max} $<$	$\rfloor_{i,1}$	$\rceil_{i,1}$	$\rfloor_{i,2}$	$\rceil_{i,2}$
1		d_i	d_i	d_{i+1}	$\gamma_i\, t_{max}$	t_{max}	t_{max}	$1-\gamma_i\,(1-t_{max})$
2		d_i	d_{i+1}		$\gamma_i\, t_{max}$	$\gamma_{i+1}\, t_{max}$	$1-\gamma_{i+1}\,(1-t_{max})$	$1-\gamma_i\,(1-t_{max})$
3	d_i	d_{i+1}	d_i	d_{i+1}	0	t_{max}	t_{max}	1
4	d_i	d_{i+1}	d_{i+1}		0	$\gamma_{i+1}\, t_{max}$	$1-\gamma_{i+1}\,(1-t_{max})$	1

Table 1: Values of the integral boundaries $\rfloor$ and $\rceil$ depending on position of T_{min} and T_{max} with respect to d_i and d_{i+1}, obtained with $\rfloor_{i,1} = \max(0, t_{i,1})$, $\rceil_{i,1} = \min(t_{i+1,1}, t_{max})$, $\rfloor_{i,2} = \max(t_{max}, t_{i+1,2})$, and $\rceil_{i,2} = \min(t_{i,2}, 1)$ and eq. (2.3.4). The values γ_i are defined by $\gamma_i = \frac{d_i - \bar{T} + \frac{\Delta}{2}}{\Delta}$.

ν	f_{β_i}	f_{α_i}	k_1	k_2
1	$1-\gamma_i$	$0.5(1-\gamma_i^2)$	0	1
2	$\gamma_{i+1}-\gamma_i$	$0.5(\gamma_{i+1}^2-\gamma_i^2)$	1	1
3	1	0.5	0	0
4	γ_{i+1}	$0.5\gamma_{i+1}^2$	1	0

Table 2: Values of f_{α_i} and f_{β_i} of $\widetilde{DEP}_i$ depending on the combination of T_{min} and T_{max} obtained by substituting the values for $\rfloor$ and $\rceil$ of tab. 1 in eq. (2.3.3). The values γ_i are defined by $\gamma_i = \frac{d_i - \bar{T} + \frac{\Delta}{2}}{\Delta}$.

In the following we transform the resulting function for $\widetilde{DEP}_i$ (cf. (2.3.3)) to a general polynomial form which is more convenient for the numerical evaluation and particularly for the subsequent evaluation of the expected value.

The solution for the general ν^{th} case of eq. (2.3.3) can be expressed by

$$\widetilde{DEP}_{i,\nu}(k_1, k_2, \bar{T}, \Delta) = \begin{cases} \beta_i((1-k_1) + k_1\gamma_{i+1} - k_2\gamma_i) + \\ \alpha_i \frac{\Delta}{2}((1-k_1) + k_1\gamma_{i+1}^2 - k_2\gamma_i^2), & d_i - \frac{\Delta}{2} \le \bar{T} \le d_{i+1} + \frac{\Delta}{2} \\ 0 & , \text{else} \end{cases} \tag{2.3.5}$$

$$\text{with} \quad \alpha_i = \frac{\widetilde{dep}(d_{i+1}) - \widetilde{dep}(d_i)}{d_{i+1} - d_i}, \; \beta_i = \widetilde{dep}(d_i) + \alpha_i \left(\bar{T} - \frac{\Delta}{2} - d_i\right), \; \gamma_i = \frac{d_i - \bar{T} + \frac{\Delta}{2}}{\Delta}.$$

The parameters k_1 and k_2 (cf. tab. 2) depend thereby on the combination of $\bar{T}$ and Δ which differ for the four cases $\nu = 1, \ldots, 4$ (cf. fig. 3a).

By backsubstitution of γ_i and β_i $\widetilde{DEP}_{i,\nu}(k_1, k_2, \bar{T}, \Delta)$ leads to a polynomial in $\bar{T}$ and a rational function in Δ, which can be expressed (e.g. with the help of symbolic calculation software) by

$$\widetilde{DEP}_{i,\nu}(k_1, k_2, \bar{T}, \Delta) = \sum_{j=1}^{3} \sum_{l=1}^{3} \xi_{j,l}(k_1, k_2) \bar{T}^{l-1} \Delta^{j-2} \tag{2.3.6}$$

with the elements $\xi_{j,l}(k_1, k_2)$ of the coefficient matrix

$$\xi_{j,l}(k_1, k_2) \in C_D = \begin{pmatrix} \varrho_i(d_{i+1}k_1 - d_i k_2) + \alpha_i(\frac{d_{i+1}^2 k_1}{2} - \frac{d_i^2 k_2}{2}) & \varrho_i(-k_1 + k_2) & \frac{\alpha_i(-k_1+k_2)}{2} \\ \varrho_i(1 - \frac{3k_1}{2} + \frac{k_2}{2}) + \alpha_i(d_i k_2 - d_{i+1}k_1) & \frac{\alpha_i(2-k_1-k_2)}{2} & 0 \\ \frac{3\alpha_i(k_1-k_2)}{8} & 0 & 0 \end{pmatrix}$$

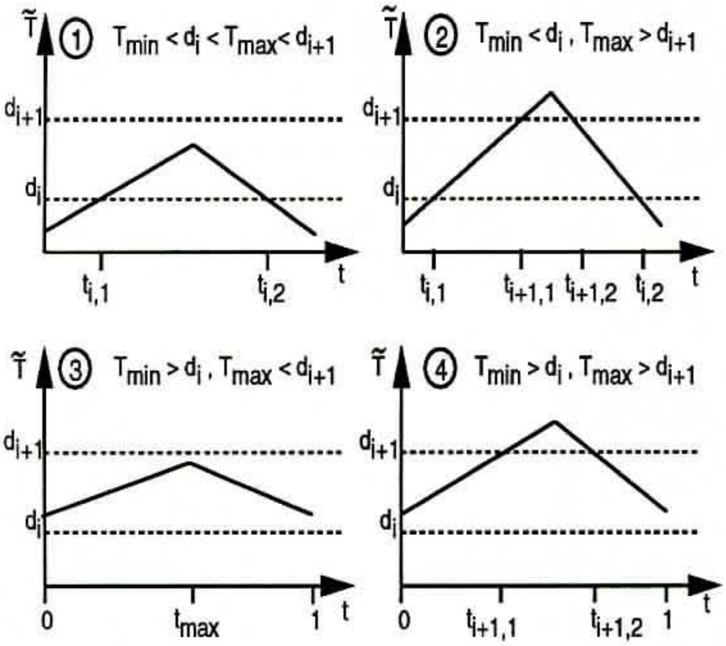

Figure 2: Four different cases of daily temperature triangles, defined through the position of the temperature extrema relative to the threshold values d_i and d_{i+1} of the dependence function approximation. The temperature approximation reaches the thresholds d_i resp. d_{i+1} at times $t_{i,1}$ resp. $t_{i+1,1}$ in the increasing part of the triangle, and at the times $t_{i,2}$ resp. $t_{i+1,2}$ in the decreasing part of the triangle.

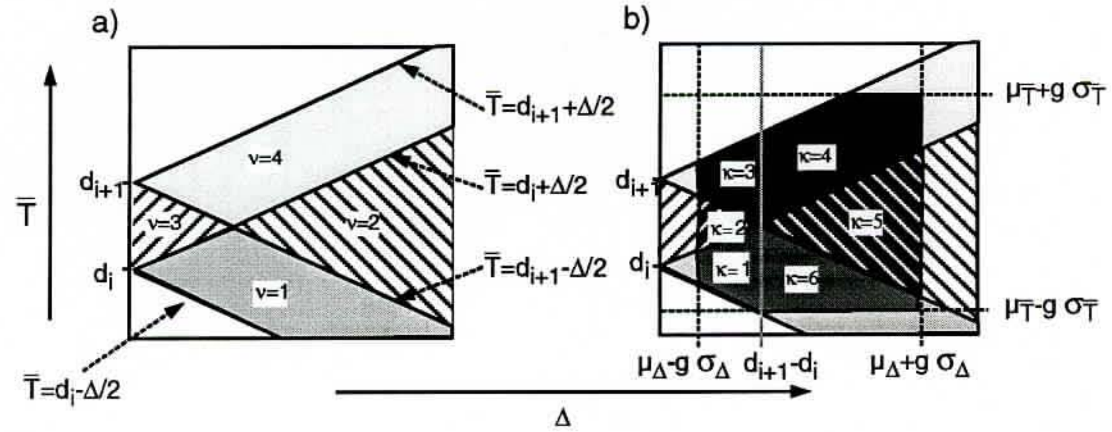

Figure 3: **a)** Combinations of daily temperature amplitude Δ and daily temperature mean $\bar{T}$, which determine four different possibilities (areas Ω_ν, $\nu = 1, \dots, 4$) of the position of the daily temperature triangle relative to the temperature thresholds d_i and d_{i+1} (cf. fig. 2), and hereby of the formulation of the daily dependence function $\widetilde{DEP}_i$ in eq. (2.3.6). **b)** Final integration area in the $(\bar{T}, \Delta)$-plane: It consists of the six areas Ω_κ where the expected value of the daily dependence function is evaluated, i.e. the double integral (2.3.10) is solved. The four areas of a) are further bounded by the values of temperature means $\mu_{\bar{T}} \pm 2\sigma_{\bar{T}}$ and amplitudes $\mu_\Delta \pm 2\sigma_\Delta$, outside of which the density function approximation is 0 (cf. fig. 1c). The areas Ω_1 and Ω_4 of a) are split by the line $\Delta = d_{i+1} - d_i$.

(2.3.7)

and $\varrho_i = dep(d_i) - \alpha_i d_i$.

Example 2.1 *The case of fig. 2.2 yields*

$$T_{max} = \bar{T} + \frac{\Delta}{2} > d_{i+1} \Rightarrow \bar{T} > d_{i+1} - \frac{\Delta}{2},$$

$$T_{min} = \bar{T} - \frac{\Delta}{2} < d_i \Rightarrow \bar{T} < d_i + \frac{\Delta}{2},$$

and corresponds thus in fig. 3a) to area Ω_2. *Therefore, from tab. 2 we get the values* $f_{\beta_i} = \gamma_{i+1} - \gamma_i, f_{\alpha_i} = 0.5(\gamma_{i+1}^2 - \gamma_i^2), k_1 = 1$, *and* $k_2 = 1$. *With these values, the* i^{th} *part of the approximated dependence function yields*

$$\begin{aligned}\widetilde{DEP}_{i,2}(1,1,\bar{T},\Delta) &= \beta_i(\gamma_{i+1}-\gamma_i) + 0.5\alpha_i\Delta(\gamma_{i+1}^2-\gamma_i^2) && \textit{in representation of eq. (2.3.3)}\\ &= \frac{1}{\Delta}\left(\varrho_i(d_{i+1}-d_i) + 0.5\alpha_i(d_{i+1}^2-d_i^2)\right) + \alpha_i(d_i - d_{i+1}) && \textit{as polynomial (cf. eq. (2.3.6)).}\end{aligned}$$

Expected value Now, with the approximated daily dependence function integral $\widetilde{DEP}_i$ we are able to determine the expected values by using the approaches DAT (2.1.6), EDDT1, and EDDT2 (2.1.7). (1) For approach DAT, the arguments $\bar{T}$ and Δ in eq. (2.3.6) are replaced by their mean values $\mu_{\bar{T}}$ and μ_Δ. Analogously to eq. (2.1.6) we get

$$E[\widetilde{DEP}_i] \simeq \widetilde{DEP}_{i,\nu}(k_1, k_2, \mu_{\bar{T}}, \mu_\Delta). \tag{2.3.8}$$

The number ν and the values k_1 and k_2 depend on the position of $\mu_{\bar{T}}$ and μ_Δ in the $(\bar{T},\Delta)$-plane relatively to the actual values of d_i and d_{i+1} (cf. fig. 3a) and tab. 2). To obtain the total expected values, the $(E[\widetilde{DEP}_i])_{i>0}$ have to be summed over all i, i.e.

$$E[\widetilde{DEP}] = \sum_i E[\widetilde{DEP}_i] \simeq \sum_i \widetilde{DEP}_{i,\nu(i)}(k_1, k_2, \mu_{\bar{T}}, \mu_\Delta) \tag{2.3.9}$$

(2) For approaches EDDT1 and EDDT2 we substitute in eq. (2.1.7) the daily temperature dependence integral $DEP(\bar{T},\Delta)$ and the probability densities $p_{\bar{T}}(y)$ and $p_\Delta(z)$ by their approximations $\widetilde{DEP}_i$ (2.3.6), $\widetilde{p}_{\bar{T}}(y)$, and $\widetilde{p}_\Delta(z)$ (2.2.3) and obtain

$$E[\widetilde{DEP}_i] \simeq \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \widetilde{DEP}_i(k_1, k_2, y, z)\, \widetilde{p}_{\bar{T}}(y)\, dy\ \widetilde{p}_\Delta(z)\, dz. \tag{2.3.10}$$

Because $\widetilde{DEP}_i$ is different in the four areas $\Omega_\nu, \nu = 1, \ldots, 4$ (cf. eq. (2.3.5)) in the $(\bar{T},\Delta)$-plane as shown in fig. 3a), we now have to solve the resulting integral over each of these four domains. These integration domains are furtherly bounded by the values $\mu_{\bar{T}} \pm 2\sigma_{\bar{T}}$ and $\mu_\Delta \pm 2\sigma_\Delta$ (depending on $\bar{T}$ and Δ) which define the interval $[\mu_x - 2\sigma_x, \mu_x + 2\sigma_x]$ where the approximation of the density functions is $\neq 0$ (cf. fig. 1c). Furthermore, the areas Ω_1 and Ω_4 are split by the line $\Delta = d_{i+1} - d_i$ to obtain as integration boundaries of the inner integral continuous functions of the outer integration variable z. The so resulting six integration domains $\Omega_\kappa, \kappa = 1, \ldots, 6$ are shown in fig. 3b). The resulting boundaries together with the values k_1 and k_2 are listed in tab. 3. For the approximation of the expected value we get now

$$\begin{aligned}E[\widetilde{DEP}_i] &\simeq \underbrace{\sum_{\kappa=1}^{6} \iint_{\Omega_\kappa} \underbrace{\widetilde{DEP}_i(k_1, k_2, y, z)\, \widetilde{p}_{\bar{T}}(y)\, \widetilde{p}_\Delta(z)}_{P_\kappa} dy\ dz}_{\widetilde{E}_\kappa[\widetilde{DEP}_i]}\\ &= \sum_{\kappa=1}^{6} \int_{\rfloor_{\Delta,\kappa}}^{\rceil_{\Delta,\kappa}} \int_{\rfloor_{\bar{T},\kappa}}^{\rceil_{\bar{T},\kappa}} P_\kappa\, dy\, dz.\end{aligned}$$

κ	$\rfloor_{\Delta,\kappa}$	$\rceil_{\Delta,\kappa}$	$\rfloor_{\bar{T},\kappa}$	$\rceil_{\bar{T},\kappa}$	k_1	k_2
1	$\max(0, \vdash_\Delta)$	$\min(d_{i+1}-d_i, \dashv_\Delta)$	$\max(d_i - \frac{\Delta}{2}, \vdash_{\bar{T}})$	$\min(d_i + \frac{\Delta}{2}, \dashv_{\bar{T}})$	0	1
2	$\max(0, \vdash_\Delta)$	$\min(d_{i+1}-d_i, \dashv_\Delta)$	$\max(d_i + \frac{\Delta}{2}, \vdash_{\bar{T}})$	$\min(d_{i+1} - \frac{\Delta}{2}, \dashv_{\bar{T}})$	0	0
3	$\max(0, \vdash_\Delta)$	$\min(d_{i+1}-d_i, \dashv_\Delta)$	$\max(d_{i+1} - \frac{\Delta}{2}, \vdash_{\bar{T}})$	$\min(d_{i+1} + \frac{\Delta}{2}, \dashv_{\bar{T}})$	1	0
4	$\max(d_{i+1}-d_i, \vdash_\Delta)$	$\dashv_\Delta$	$\max(d_i - \frac{\Delta}{2}, \vdash_{\bar{T}})$	$\min(d_i + \frac{\Delta}{2}, \dashv_{\bar{T}})$	0	1
5	$\max(d_{i+1}-d_i, \vdash_\Delta)$	$\dashv_\Delta$	$\max(d_i + \frac{\Delta}{2}, \vdash_{\bar{T}})$	$\min(d_{i+1} - \frac{\Delta}{2}, \dashv_{\bar{T}})$	1	1
6	$\max(d_{i+1}-d_i, \vdash_\Delta)$	$\dashv_\Delta$	$\max(d_{i+1} - \frac{\Delta}{2}, \vdash_{\bar{T}})$	$\min(d_{i+1} + \frac{\Delta}{2}, \dashv_{\bar{T}})$	1	0

Table 3: The integration boundaries $\rfloor_{\Delta,\kappa}$, $\rceil_{\Delta,\kappa}$, $\rfloor_{\bar{T},\kappa}$, and $\rceil_{\bar{T},\kappa}$ and the values k_1 and k_2 depend on the integration domains Ω_κ, which are defined by the combination of Δ and $\bar{T}$ (cf. fig. 3) and by the boundaries of the parabola approximating the density functions of $\bar{T}$ and Δ (cf. fig. 1c).
We use the abbreviations $\vdash_\Delta = \mu_\Delta - g \cdot \sigma_\Delta$, $\dashv_\Delta = \mu_\Delta + g \cdot \sigma_\Delta$, $\vdash_{\bar{T}} = \mu_{\bar{T}} - g \cdot \sigma_{\bar{T}}$, $\dashv_{\bar{T}} = \mu_{\bar{T}} + g \cdot \sigma_{\bar{T}}$.

Thereby, the integrand P_κ is a polynomial of 4^{th} order in y and of 3^{rd} order in z because the density function approximations $\widetilde{p}_{\bar{T}}(y)$ and $\widetilde{p}_\Delta(z)$ (2.2.3) can be written as polynomials

$$\widetilde{p}_{\bar{T}}(y) = \sum_{n=1}^{3} \zeta_n y^{n-1}, \quad \widetilde{p}_\Delta(z) = \sum_{m=1}^{3} \rho_m z^{m-1}$$

with the coefficients

$$\zeta_n \in C_{\bar{T}} = \begin{pmatrix} \frac{3}{4g\sigma_{\bar{T}}} - \frac{3\mu_{\bar{T}}^2}{4g^3\sigma_{\bar{T}}^3} \\ \frac{3\mu_{\bar{T}}}{2g^3\sigma_{\bar{T}}^3} \\ \frac{-3}{4g^3\sigma_{\bar{T}}^3} \end{pmatrix} \quad \text{and} \quad \rho_m \in C_\Delta = \begin{pmatrix} \frac{3}{4g\sigma_\Delta} - \frac{3\mu_\Delta^2}{4g^3\sigma_\Delta^3} \\ \frac{3\mu_\Delta}{2g^3\sigma_\Delta^3} \\ \frac{-3}{4g^3\sigma_\Delta^3} \end{pmatrix}$$

and therefore we can write

$$\begin{aligned} P_\kappa &= \widetilde{DEP}_i(k_1, k_2, y, z)\widetilde{p}_{\bar{T}}(y)\widetilde{p}_\Delta(z) \\ &= \left(\sum_{j=1}^{3}\sum_{l=1}^{3} \xi_{j,l}(k_1,k_2) y^{l-1} z^{j-2} \right) \left(\sum_{n=1}^{3} \zeta_n y^{n-1} \right) \left(\sum_{m=1}^{3} \rho_m z^{m-1} \right) \\ &= \sum_{j,l,m,n=1}^{3} \underbrace{\xi_{j,l}(k_1,k_2)\rho_m \zeta_n}_{c_{j,l,m,n}(k_1,k_2)} y^{l+n-2} z^{j+m-3} \\ \Rightarrow E[\widetilde{DEP}_i] &\simeq \sum_{\kappa=1}^{6} \int_{\rfloor_{\Delta,\kappa}}^{\rceil_{\Delta,\kappa}} \int_{\rfloor_{\bar{T},\kappa}}^{\rceil_{\bar{T},\kappa}} \sum_{j,l,m,n=1}^{3} c_{j,l,m,n}(k_1,k_2) y^{l+n-2} z^{j+m-3} dy \; dz. \end{aligned} \tag{2.3.11}$$

This integral can be solved with some calculation effort, e.g. with the help of symbolic calculation software, because the integrand as well as the bounds of the inner integral are polynomials. An example is given at the end of this section.
Summarized, the expected value of $dep(T)$ is determined by summing the expected values of each of the linear pieces of the dependence function. The expected value of the i^{th} linear piece is calculated by first determining the thresholds d_i and d_{i+1} of this piece. Then the integrals over each of the areas Ω_κ have to be solved and summed. For each area, κ determines the values k_1 and k_2 and the integration borders $\rfloor_{\Delta,\kappa}$, $\rceil_{\Delta,\kappa}$, $\rfloor_{\bar{T},\kappa}$, and $\rceil_{\bar{T},\kappa}$ (cf. tab. 3). With the k-values, the coefficients $\xi_{j,l}(k_1,k_2)$ can now be determined from matrix C_D (2.3.7) and with this information, the double integral (2.3.11) can be solved.
The following example explains this procedure for $\kappa = 5$.

Example 2.2 *For the case $\kappa = 5$, which corresponds to fig. 2.2, we get with $d_{i+1} - d_i > \mu_\Delta - g \cdot \sigma_\Delta$, $d_i + \frac{\Delta}{2} > \mu_T - g \cdot \sigma_T$, and $d_{i+1} - \frac{\Delta}{2} < \mu_T + g \cdot \sigma_T$, from tab. 3 the values $k_1 = 1$, $k_2 = 1$, $\rfloor_{\Delta,5} = d_{i+1} - d_i$, $\rceil_{\Delta,5} = \mu_\Delta + g \cdot \sigma_\Delta$, $\rfloor_{\bar{T},5} = d_i + \frac{\Delta}{2}$ and $\rceil_{\bar{T},5} = d_{i+1} - \frac{\Delta}{2}$. Hence the 5^{th} part of eq. 2.3.11 is given by*

$$\widetilde{E}_5[\widetilde{DEP}_i] = \int_{d_{i+1}-d_i}^{\mu_\Delta + g\sigma_\Delta} \int_{d_i + \frac{\Delta}{2}}^{d_{i+1} - \frac{\Delta}{2}} \sum_{j,l,m,n} c_{j,l,m,n}(1,1) y^{l+n-2} z^{j+m-3} dy \, dz$$

$$
\begin{aligned}
&= \int_{d_{i+1}-d_i}^{\mu_\Delta+g\sigma_\Delta} \sum_{j=1}^{3}\sum_{m=1}^{3} \rho_m\ z^{j+m-3} \int_{d_i+\frac{z}{2}}^{d_{i+1}-\frac{z}{2}} \sum_{l=1}^{3}\sum_{n=1}^{3} \zeta_n \xi_{j,l}(1,1)\ y^{l+n-2} dy\, dz \\
&\quad \textit{because only } \xi_{1,1},\ \xi_{2,1} \neq 0 \textit{ we get thus} \\
&= \int_{d_{i+1}-d_i}^{\mu_\Delta+g\sigma_\Delta} \sum_{j=1}^{2}\sum_{m=1}^{3} \rho_m\ z^{j+m-3}\xi_{j,1}(1,1) \int_{d_i+\frac{z}{2}}^{d_{i+1}-\frac{z}{2}} \sum_{n=1}^{3} \zeta_n\ y^{1+n-2} dy\, dz \\
&= \int_{d_{i+1}-d_i}^{\mu_\Delta+g\sigma_\Delta} \sum_{j=1}^{2}\sum_{m=1}^{3} \rho_m\ z^{j+m-3}\xi_{j,1}(1,1) \sum_{n=1}^{3} \frac{\zeta_n}{n} \left((d_{i+1}-\frac{z}{2})^n - (d_i+\frac{z}{2})^n\right) dz \\
&= \int_{d_{i+1}-d_i}^{\mu_\Delta+g\sigma_\Delta} \sum_{j=1}^{2}\sum_{m=1}^{3} \rho_m\ z^{j+m-3}\xi_{j,1}(1,1) \cdot \\
&\quad \Bigg(\underbrace{\left(\zeta_1 (d_{i+1}-d_i) + \frac{\zeta_2}{2}(d_{i+1}^2 - d_i^2) + \frac{\zeta_3}{3}(d_{i+1}^3 - d_i^3)\right)}_{a_0} + \\
&\quad \underbrace{\left(-\zeta_1 + \frac{\zeta_2}{2}(-d_{i+1}-d_i) + \frac{\zeta_3}{2}(-d_{i+1}^2 - d_i^2\right)}_{a_1} z + \\
&\quad \underbrace{\frac{\zeta_3}{4}(d_{i+1}-d_i)\Big)}_{a_2} z^2 + \underbrace{\frac{\zeta_3}{12}}_{a_3} z^3 \Bigg)\, dz \\
&= \int_{d_{i+1}-d_i}^{\mu_\Delta+g\sigma_\Delta} \sum_{j=1}^{2}\sum_{m=1}^{3} \rho_m \xi_{j,1}(1,1) \cdot \sum_{n=0}^{3} a_n z^{j+m+n-3}\, dz \\
&= \sum_{j=1}^{2}\sum_{m=1}^{3}\sum_{n=0}^{3} \frac{\rho_m \xi_{j,1}(1,1) a_n}{j+m+n-2} \cdot \left((\mu_\Delta + g\sigma_\Delta)^{j+m+n-2} - (d_{i+1}-d_i)^{j+m+n-2}\right).
\end{aligned}
$$

3 Properties of Resulting Methods

Table 4 gives an overview of the eight different methods, which have been derived in this paper, with respect to their temporal resolution, their data requirements and the approximations they use.
The methods differ particularly in the way in which they use the information about the temperature variability contained in the input data.
Method EDH takes into account the intra daily variability by using hourly input data. Methods EDDT1, EDHT1, EDDT2, EDHT2, and DAT extract the information about the intra daily variability from daily temperature amplitudes by assuming a triangle-shaped temperature course, which is either used to estimate the statistical parameters of the hourly temperatures or to calculate the daily dependence function and its expected value. For the case that only daily (method EDM) or even long term means (method DA) are available, intra daily variability is neglected.

4 Discussion

In this paper, a range of new approaches for aggregating temperature dependence functions to longer time periods have been derived. The methods are constructed for a variety of input data resolutions and allow the inclusion of temporal temperature variability in ecological models. Table 4 gives an overview of these methods, their temporal resolution, input data needs and any approximations used. Thus, an appropriate method now can be chosen from this set, depending on the available input data, the needed aggregation period, and the necessary precision .

Method	Abbr.	Type	Time resolution	Input data Variables	Statistical parameters	exact formula	Approximation	aprox. formula
Expectation value dependence function of hourly temperatures	EDH	E	hours	T	μ_T, σ_Δ	2.1.3	dep	2.3.1
Expectation value of dependence function of hourly temperatures approx. by triangle based on mean and amplitude	EDHT1	E	days	$\bar{T}$, $\Delta = T_{max} - T_{min}$	$\mu_{\bar{T}}, \sigma_{\bar{T}}$, $\mu_\Delta, \sigma_\Delta$	2.1.3	dep, TC	2.3.1, 2.3.2
Expectation value of dependence function of hourly temperature approx. by triangle based on mean and amplitude	EDDT1	E	days	$\bar{T}$, $\Delta = T_{max} - T_{min}$	$\mu_{\bar{T}}, \sigma_{\bar{T}}$, $\mu_\Delta, \sigma_\Delta$	2.1.7	dep, TC, ND	2.3.11
Expectation value of dependence function of hourly temperatures approx. bytriangle based on extrema	EDHT2	E	days	$\bar{T}_m = \frac{T_{max}+T_{min}}{2}$ $\Delta = T_{max} - T_{min}$	$\mu_{\bar{T}_m}, \sigma_{\bar{T}_m}$ $\mu_\Delta, \sigma_\Delta$	2.1.3	dep, TC	2.3.1, 2.3.2
Expectation value of dependence function of daily temperature triangle based on extrema	EDDT2	E	days	$\bar{T}_m = \frac{T_{max}+T_{min}}{2}$ $\Delta = T_{max} - T_{min}$	$\mu_{\bar{T}_m}, \sigma_{\bar{T}_m}$ $\mu_\Delta, \sigma_\Delta$	2.1.7	dep, TC, ND	2.3.11
Expectation value of dependence function of daily temperature mean	EDM	E	days	$\bar{T}$	$\mu_{\bar{T}}, \sigma_\Delta$,	2.1.4	dep	2.3.1
Dependence function of average daily temperature triangle	DAT	A	months	$\mu_{\bar{T}}, \mu_\Delta$	$\mu_{\bar{T}}, \mu_\Delta$	2.1.6	dep, TC	2.3.8
Dependence function of average temperature	DA	A	months	$\mu_{\bar{T}} = \mu_T$	$\mu_{\bar{T}}$	2.1.1	dep	2.1.1

Table 4: Overview over the temperature dependence aggregation methods: They are divided according to the type of method (explicit expectation value calculation, or dependence function of average input), the resolution and kind of the needed input data (T: hourly temperature, T_{max}, T_{min}: daily temperature extrema, Δ: daily temperature amplitude, $\bar{T}$: daily temperature mean, $\bar{T}_m$: approximated daily temperature mean, $\mu_{\bar{T}}$: monthly mean temperature, σ: monthly mean amplitude), the statistical parameters estimated from these data (μ: mean and σ: standarddeviation) and the approximations used (dep: dependence function, TC: daily temperature course, ND: normal distribution). References to the formulae are given in columns "exact formula" and "approx. formula". Note, the method DA is a widly used approach.

The main characteristics and differences of the methods are:
(1) Method EDH takes into account the intra daily variability by using hourly input data and hence including the temperature variance. **(2)** Methods EDDT1, EDHT1, EDDT2, EDHT2, and DAT extract the information about the intra daily variability from daily temperature amplitudes by assuming a triangle-shaped temperature course, which is either used to estimate the statistical parameters of the hourly temperatures or to calculate the daily dependence function and its expected value. **(3)** Method EDM uses the inter-daily variability by the variances of daily mean temperatures, but neglects the intra daily variability.
Thus, all of the presented approaches are able, to different extents, to include temporal temperature variability, in contrast to the widely used application of the dependence function to (long term) temperature means (approach DA in tab. 4).

However, the new methods might have a certain bias arising from the used approximations and assumptions. They assume the temperature variables to be normally distributed and temperature mean and amplitude to be independent of each other, which is probably not always correct. The assumption of a daily triangle temperature course similar to the triangulation method of Lindsey and

Newman (1956), might also appear crude. However, physiological time calculated with this triangulation can be sufficiently precise, if the times of the daily temperature maxima are known, as shown for the example of codling moth development (LISCHKE 1991). Thus for an adequate use of the triangle approximation either the temperature maximum time is required for each day or a method which calculates the daily dependence function based on the triangulation independently of this time.
The latter holds for the methods EDDT1 and EDDT2, where the temperature maximum time drops out during the calculation of the daily temperature dependence. This could be an advantage over the methods EDHT1 and EDHT2 and also over the sine-sine-method of Allen (1976), because in their the daily dependence approximation this time still appears, and hence has to be estimated e.g. to be at noon.

To assess the effects of the aforementioned potential biases and the applicability of the presented methods, the precision and efficiency of the methods have been tested (LISCHKE ET AL. 1995B) in several ecological applications and compared to other common methods. The tests revealed that it can be crucial to use all available variability information dependent on the precision requirements to obtain satisfying results. Also, the approaches EDH, EDHT1, and EDHT2 combined high precision with high speed on their respective levels of resolution. The effect of the bias introduced by assuming the temperature maximum to occur at noon in EDHT1 and EDHT2 turned out to be negligible.

The presented methods can be used in a wide range of ecological models where variable abiotic factors are affecting the dynamics, e.g. in pest prognosis models. They can be particularly useful where dynamics which still depend on smaller time scale variations have to be simulated on large time scales, as e.g. weather dependent plant growth in dynamic vegetation models which are used to assess the impact of climate change over centuries. For instance, the forest succession model FORCLIM reacts very sensitively (FISCHLIN ET AL. 1994) to whether the climate input is formulated as constant input or by a stochastic weather generator on the monthly scale but runs for several hundred years. Another example are models for the simulation of the forest carbon cycle as reviewed by Perruchoud and Fischlin (1995), which depend on temperature and run for even longer simulation periods.

The construction of the approaches is not restricted to the specific approximations we presented here, other ones could be chosen as e.g. quadratic polynomials for the daily temperature course, exponential functions to approximate the temperature dependence function, or piecewise linear polynomials to approximate density functions. The latter could extend the range of applicability also to other than normal distributions, even to empirical ones.
The approaches are also not restricted to dependence functions of temperature. The methods EDH and EDM which do not assume a certain daily temperature course can also be applied to dependence functions of other abiotic factors, or more generally to the calculation of arbitrary functions of normally distributed random variables. We used e.g. the method EDH successfully to calculate the expected values of a nonlinear light dependence function in the forest dynamics model DISCFORM (LISCHKE ET AL. 1995A).
The concept of approximating the daily temperature course, which is the basis of the methods DAT, EDHT1, EDHT2, EDDT1, and EDDT2 could be transferred to other periodicities, as e.g. interdecadal temperature oscillations (MANN ET AL. 1995) or the yearly temperature course. This would allow the estimation of long term dependence functions of monthly temperature means, given yearly statistic parameters of extrema and means of daily or monthly temperature means.
The methods are even not restricted to temporal variability. It is possible to also apply them for spatially varying input variables, e.g. during an spatial model upscaling.

5 Conclusions

Now we have a variety of methods at hand, which can be applied to every temperature dependence function by simple linearisation. They are suitable for different temperature input data resolutions, e.g. minutely or hourly temperature, daily mean and daily amplitude, daily extrema, monthly mean and monthly mean day-amplitude and monthly mean. With these methods it is possible to use as much information about the variability in the input data as available through daily amplitudes or standard deviations of hourly temperatures, and can be used for arbitrarily large time steps ranging from days to millenia. Finally they can be applied to any kind of dependence function in many fields of ecological modelling applications.

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A Overview of the used symbols

Table 5: Table of Symbols

Symbol	Meaning	Unit
t	time	$days$
$T(t)$	temperature at time t	^0C
$\tilde{T}(t)$	approximation of temperature at time t	^0C
$\bar{T}$	daily temperature average	^0C
T_{min}	daily minimum temperature	^0C
T_{max}	daily maximum temperature	^0C
t_{max}	time of daily maximum temperature	$days$
Δ	daily temperature amplitude	^0C
$dep(T)$	temperature dependence function	-
$\widetilde{dep}(T)$	approximation of temperature dependence function	-
$\widetilde{dep}_i(T)$	approximation for ith linear part of temperature dependence function	-
d_i	discretization of $\widetilde{dep}(T)$, lower temperature threshold of $\widetilde{dep}_i(T)$	^0C
d_{i+1}	upper temperature threshold of $\widetilde{dep}_i(T)$	^0C
$E[X]$	expected value of random variable X	same as X
$p_X(X)$	density function of random variable X	-
$\mu_X(X)$	mean of random variable X	same as X
$\sigma_X(X)$	standarddeviation of random variable X	same as X
$\tilde{p}_X(X)$	approximation of density function of random variable X	-
ζ_n, ρ_m	coefficients of $\tilde{p}_{\bar{T}}(y)$ and $\tilde{p}_\Delta(z)$ in polynomial form	
$\vdash, \dashv$	boundaries of interval where $\tilde{p}_X(x) \neq 0, X = \bar{T}, \Delta$	^0C
DEP	daily temperature dependence function integral	-
$\widetilde{DEP}_{i,\nu}(k_1, k_2, \bar{T}, \Delta)$	daily integral over ith linear part of approximation of temperature dependence function	-
Ω	integration area	-
$\rfloor, \rceil$	integration boundaries due to position of T_{min} and T_{max} relatively to d_i and d_{i+1}	$days, ^0C$
$i = 1, ..., m$	index of dependence function discretization	-
ν, κ	indices of subintegrals and integration areas	-
$\alpha, \beta, \gamma,$ f_{alpha}, f_{beta}	variables used for substitution	-
$\xi_{j,l}(k_1, k_2) \in C_D$	coefficients of $\widetilde{DEP}_i(k_{1,\nu}, k_{2,\nu})$ in polynomial form	-

References

Aceituno, P. 1979. Statistical formula to estimate heating or cooling degree-days. Agric.For.Meteorol. 20:227–232.

Allen, J. 1976. A modified sine wave method for calculating degree days. Environ.Ent. 5:388–396.

Baskervile, G., and P. Emin. 1969. Rapid estimation of heat accumulation from maximum and minimum temperatures. Ecology.

Blago, N., and E. Dickler. n.d. Effectiveness of the Californian prognosis model "BUGOFF2" for Cydia pomonella L. (Lepidoptera, Tortricidae) under Central European conditions.

Bugmann, H. 1994. On the ecology of mountainous forests in a changing climate: A simulation study. PhD thesis. Swiss Federal Institute of Technology Zuerich. Diss. No. 10638.

Fischlin, A., H. Bugmann, and D. Gyalistras. 1994. Sensitivity of a forest ecosystem model to climate parametrization schemes. Env. Poll. 87:267–282.

Kirsta, Y. B., and V. Tarabrin. 1994. Real biological time and its calculation in wheat. Ecol.Modelling.

Lindsey, A., and J. Newman. 1956. Use of official weather data in spring time - Temperature analysis of an Indiana phenological record. Ecology 37:812–823.

Lischke, H. 1991. Ein Modell zur Simulation der Populationsdynamik des Apfelwicklers (Cydia Pomonella L. (Lepidoptera, Tortricidae). PhD thesis. University of Heidelberg.

Lischke, H. 1992. A model to simulate the population dynamics of the codling moth (Cydia pomonella L.(Lepidoptera, Torticidae)): Parameter estimation and sensitivity analysis. Acta Hort. 313:331–338.

Lischke, H., and N. Blago. 1990. A model to simulate the population dynamics of the codling moth (Cydia pomonella L.(Lepidoptera, Torticidae)): Development and male moth flight. Acta Hort. 276:43–52.

Lischke, H., T. J. Loeffler, and A. Fischlin. 1996a. How to simplify individual based forest succession models? - Aggregating patches to a stack of forest discs and trees to populations with random dispersions. Technical Report 28. Systems Ecology, ETH Zurich, Switzerland.

Lischke, H., T. Loeffler, and A. Fischlin. 1996b. Calculating temperature dependence over long time periods: A comparison of methods. Technical Report 27. Systems Ecology, Institute of Terrestrial Ecology. Grabenstr.11A, CH-8952 Schlieren, Switzerland.

Mann, M., J. Park, and R. Bradley. 1995. Global interdecadal and century-scale climate oscillations during the past five centuries. 378:266–270.

Parton, W., and J. A. Logan. 1981. A model for diurnal variation in soil and air temperature. Agric.Meteorol. 23:205–216.

Perruchoud, D., and A. Fischlin. 1995. The response of the carbon cycle in undisturbed forest ecosystems to climate change: A review of plant-soil models. 22:2603–2618.

Prentice, I., M. Sykes, and W. Cramer. 1993. A simulation model for the transient effects of climate change on forest landscapes. Ecol.Modelling 65:51–70.

Sharpe, P., and D. DeMichele. 1977. Reaction kinetics of poikilotherm development. J.Theor.Biol. 64:649–670.

Stinner, R., A. Gutierrez, and G. Butler. 1974. An algorithm for temperature-dependent growth rate simulation. Can.Ent. 106:519–524.

Wagner, T., H. Wu, P. Sharpe, R. Schoolfield, and R. Coulson. 1984. Modeling insect development rates: A literature review and aapplication of a biophysical model. Ann.Ent.Soc.Am. 77:208–221.

Worner, S. 1988. Evaluation of diurnal temperature models and thermal summation in New Zealand. J.Econ.Entomol. 81:9–13.

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